Notes on Fang and Zhao's 2010 JDE paper

Remark: Theorem 4.1 and Theorem 4.2 in Fang and Zhao's paper [2] are still valid in the case where $D = 0$.

It suffices to check that [2, Theorem 4.1] is true in this case. The below are my explanations.

1. In the case $D = 0$, one can choose the Green function $\Gamma_\alpha(t, z) = e^{-\alpha t}\delta(z)$. Then we still have integral equation (4.5). But all the arguments for the spreading speed and traveling waves still work (after obvious changes). To compute c^*, we just let $D = 0$ in (4.7), which is the same as the minimal wave speed obtained intuitively by linearizing the wave profile equation at zero.

2. In the case where $D = 0$, equation (4.4) becomes equation (2.4) in paper [5]. By [5, Remark 2.1], the earlier theory is still valid in the monotone case. Consequently, the sandwich method also works for such an equation in the non-monotone case.

3. For So, Wu and Zou's model in [3] with $D = 0$, the kernel function $J(s, y)$ is as in the proof of [2, Theorem 4.2]. To computer $\mathcal{K}_J$, an easy way is to use [4, Proposition 4.2].

4. It seems that there is no need to do traveling waves for this degenerate case either by using the standard method of upper and lower solutions, or by choosing small ϵ as diffusion coefficient and then letting ϵ tend to zero (for the use of this method, see [1]).

References

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