ERRATA TO THE "SPREADING SPEEDS OF A PARTIALLY DEGENERATE REACTION-DIFFUSION SYSTEM IN A PERIODIC HABITAT" [J. DIFFERENTIAL EQUATIONS,255(2013),3983-4011]

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In the proof of [1, Lemma 2.3], the norm estimate for $v_1(t,x)$ below (2.12) is wrong. The purpose of our current note is to give a more succinct proof to correct this mistake.

Proof of Lemma 2.3. By [1, Theorem 2.1], for any given $\phi \in [0,M]_{\mathcal{C}}$, $u(t,\cdot;\phi)$ is continuous in $t \in [0,\infty)$ with respect to the compact open topology. By assumptions (A2) and (A4), it follows that there exists a cooperative 2×2 matrix A such that

$$F(x,u) - F(x,v) \leq A(u-v), \quad \forall x \in \mathbb{R},\ 0 \leq v \leq u \leq M.$$

Note that

$$u(t,\cdot) = \ell(t)u(0,\cdot) + \int_0^t \ell(t-s)F(\cdot,u(s,\cdot))\mathrm{d}s.$$

For any $\phi \geq \psi$ in $[0,M]_{\mathcal{C}}$, let $w(t,x) = u(t,x;\phi) - u(t,x;\psi)$. It then follows that

$$0 \leq w(t,\cdot) \leq \ell(t)(\phi-\psi) + \int_0^t \ell(t-s)Aw(s,\cdot)\mathrm{d}s, \quad \forall t \geq 0.$$

Since $Z(t) = e^{At}\ell(t)(\phi-\psi)$ is the unique solution of the integral equation

$$Z(t) = \ell(t)(\phi-\psi) + \int_0^t \ell(t-s)AZ(s)\mathrm{d}s,$$

the comparison principle implies that

$$0 \leq u(t,x;\phi) - u(t,x;\psi) \leq e^{At}[\ell(t)(\phi-\psi)](x), \quad \forall t \geq 0,\ x \in \mathbb{R}.$$

Let $[a,b] \subset [0,\infty)$ be a given compact interval. By the continuity of $\ell(t)\phi$ at $\phi = 0$ uniform for $t \in [a,b]$, it follows that for any $\epsilon > 0$ and $K > 0$, there are $\delta > 0$ and $H > 0$ such that

$$u_i(t,x;\phi) - u_i(t,x;\psi) < \epsilon, \quad \forall t \in [a,b],\ x \in [-K,K], i = 1,2,$$

provided that $\phi_i(x) - \psi_i(x) < \delta, \forall x \in [-H, H], i = 1, 2$. For any $\phi, \psi \in [0, M]_{\mathcal{C}}$, let $\bar{\varphi}(x) = \max\{\phi(x), \psi(x)\}$ and $\underline{\varphi}(x) = \min\{\phi(x), \psi(x)\}$. Then we have

$$|\phi_i(x) - \psi_i(x)| = \bar{\varphi}_i(x) - \underline{\varphi}_i(x), \quad \forall x \in \mathbb{R},\ i = 1, 2.$$

By the comparison principle again, it follows that

$$|u_i(t, x; \phi) - u_i(t, x; \psi)| \le u_i(t, x; \bar{\varphi}) - u_i(t, x; \underline{\varphi}), \quad \forall t \ge 0,\ x \in \mathbb{R},\ i = 1, 2,$$

and hence,

$$|u_i(t, x; \phi) - u_i(t, x; \psi)| < \epsilon, \quad \forall t \in [a, b],\ x \in [-K, K],\ i = 1, 2,$$

provided that $|\phi_i(x) - \psi_i(x)| < \delta, \forall x \in [-H, H], i = 1, 2$. Thus, $u(t, \cdot; \phi)$ is continuous in $\phi \in [0, M]_{\mathcal{C}}$ with respect to the compact open topology for t in any compact interval $[a, b] \subset [0, \infty)$. Note that for any $t, t_1 \ge 0$, $x \in \mathbb{R}$ and $\phi, \psi \in [0, M]_{\mathcal{C}}$, we have

$$|u(t, x; \phi) - u(t_1, x; \psi)| \le |u(t, x; \phi) - u(t, x; \psi)| + |u(t, x; \psi) - u(t_1, x; \psi)|.$$

This implies that $u(t, \cdot; \phi)$ is continuous in $(t, \phi) \in [0, \infty) \times [0, M]_{\mathcal{C}}$ with respect to the compact open topology.

References

[1] Chufen Wu, Dongmei Xiao, Xiao-Qiang Zhao, Spreading speeds of a partially degenerate reaction-diffusion system in a periodic habitat, *J. Diff. Eqns.*, 255(2013),3983-4011.

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