Errata to Y. Li and X.-Q. Zhao's 2025 CV PDEs paper

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In paper [1], there is a typo in (2.15): the condition $(U,V)(-\infty)=(1,0)$, $(U,V)(+\infty)=(0,1)$ should be $(U,V)(-\infty)=(0,1)$, $(U,V)(+\infty)=(1,0)$. Moreover, the statements of Theorem 2.8 are not right: the sufficient condition for $c>0$ should be either (2.16) or (2.19); and the sufficient condition for $c<0$ should be either (2.17) or (2.18). Thus, Theorem 2.8 should be revised as follows:

Theorem 2.8 *Let $(U(x-ct),V(x-ct))$ be the traveling wave solution of system (2.14) connecting $(0,1)$ to $(1,0)$. Then the following statements are valid:*

(i) If $\frac{r}{d}\geq 1$ and either $\{a_{12}\geq 2,\ 1<a_{21}\leq 1+\frac{d}{r}\}$ or $\{a_{12}\geq 1+\frac{r}{d},\ 1<a_{21}\leq 2\}$ with all the " $=$ " not holding simultaneously, then $c>0$, i.e., species v wins.

(ii) If $\frac{r}{d}\leq 1$ and either $\{a_{21}\geq 1+\frac{d}{r},\ 1<a_{12}\leq 2\}$ or $\{a_{21}\geq 2,\ 1<a_{12}\leq 1+\frac{r}{d}\}$ with all the " $=$ " not holding simultaneously, then $c<0$, i.e., species u wins.

(iii) If $r=d$, $a_{12}=a_{21}>1$, then $c=0$.

References

[1] Yan Li and Xiao-Qiang Zhao,The propagation dynamics for three species competitive-cooperative reaction-diffusion systems, *Calculus of Variations and PDEs* 64, 28(2025). https://doi.org/10.1007/s00526-024-02887-2