Notes on Hsu and Zhao's SIMA Paper

Jian Fang

Harbin Institute of Technology, China

E-mail: jfang@mun.ca

Xiao-Qiang Zhao

Memorial University of Newfoundland, Canada

E-mail: zhao@mun.ca

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Hsu and Zhao [4] showed that the spreading speed is linearly determinate and coincides with the minimal speed of monostable traveling waves for a class of non-monotone integrodifference equations. Below we make some notes on the traveling wave with the minimal speed (also called the critical wave).

1. By [4, Theorem 3.2], it follows that under appropriate assumptions on g and k, the following integrodifference equation

$$u_{n+1}(x) = \int_{\mathbb{R}} g(u_n(y))k(x-y)dy \tag{0.1}$$

has a traveling wave solution $u_n(x) = U(x+c^*n)$ with $\liminf_{\xi\to+\infty} U(\xi) \geq u^*_-$, where c^* is the spreading speed established in [5]. The proof of [4, Theorem 3.2] does not use a sequence of functions whose limits inferior at $+\infty$ lie above a fixed positive number to prove its nonuniform limit on $\mathbb{R}$ has this property. Indeed, the last sentence in the proof of [4, Theorem 3.2] clearly says that they used the property of the spreading speed to prove this asymptotic behavior by the same arguments as in [4, Theorem 3.1 (2)]. Note that [4, Theorem 3.2] also provides two sufficient conditions for $U(+\infty) = u^*$.

2. The proof of [4, Theorem 3.2] also implies that for sufficiently small $\beta > 0$, the equation (0.1) has a traveling wave solution $U(x + c^*n)$ such that $\liminf_{\xi\to+\infty} U(\xi) \geq u^*_-$, $U(0) = \beta$ and $U(x) \leq \beta, \forall x < 0$. To see this, it suffices to replace $U_j(0) = \frac{1}{2}u^*_-$ with $U_j(0) = \beta$ (after translates of wave profiles). This observation was made for integral equations in [1, Theorem 2.2 (2)].

3. In [4, Theorem 3.2], Hsu and Zhao did not prove the wave profile U with speed c^* satisfies $U(-\infty) = 0$, see the last paragraph of [4, Section 3]. Here it is worthy for us to point out that this property can be easily proved by using some arguments in the proof of [1, Lemma 3.1].

Indeed, the wave profile equation of (0.1):

$$U(x) = \int_{\mathbb{R}} g(U(y))k(x - c^* - y)dy \tag{0.2}$$

is a special case of the equation (1.3) in [1] with $c = c^*$ and $F(u, s, y) := g(u)ae^{-as}k(y + c^*s - c^*)$ for some $a > 0$. It is easy to see that the inequality (3.3) in [1] still holds if we replace $U(-\infty) = 0$ with the condition that $\limsup_{x\to-\infty} U(x)$ is small enough. This is because the condition $U(-\infty) = 0$ is only used in the proof of the inequality (3.1) in [1], which is also true if $\limsup_{x\to-\infty} U(x)$ is small enough. Clearly, the item 2 above implies that this weaker condition is satisfied, and hence, the inequality (3.3) in [1] implies that $U(-\infty) = 0$. Note that here we do not need the assumption $l_{c,\lambda} \in L^\infty(\mathbb{R})$ in [1] since we only used the inequality (3.3) in [1].

4. The similar arguments as above were employed in [2, Theorem 4.2] for lattice equations and in [3] for integral equations to prove $U(-\infty) = 0$ for critical waves.

References

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