

# Nonlinear Dynamics and Evolution Equations

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Editors

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## Preface

The papers contained in this volume of the *Fields Institute Communications* reflect a broad spectrum of current research activities on the theory and applications of nonlinear dynamics and evolution equations. They are based on lectures given during the International Conference on Nonlinear Dynamics and Evolution Equations at Memorial University of Newfoundland, St. John's, NL, Canada, July 6–10, 2004. The aim of that conference was to bring together leading experts, researchers, and graduate students for five days of high-level lectures and informal research interactions; this was made possible by generous financial support from the Centre de recherches mathématiques (CRM), Montréal, the Fields Institute for Research in Mathematical Science, Toronto, the Pacific Institute for the Mathematical Sciences (PIMS), Vancouver, and the Atlantic Association for Research in the Mathematical Sciences (AARMS).

This volume contains thirteen invited (and refereed) papers. Nine of these are *survey papers*, introducing the reader to, and describing the current state of the art in, major areas of dynamical systems, ordinary, functional and partial differential equations, and applications of such equations in the mathematical modelling of various biological and physical phenomena. These papers are complemented by four *research papers* that examine particular problems in the theory of dynamical systems (asymptotic properties of systems that are comparable to quasi-monotone systems; smoothness of center manifolds for state-dependent delay differential equations; dynamics of the asymmetric generalized May-Leonard model of three species competition; exact Poisson structures on manifolds).

Four of the survey papers deal with various aspects of the *theory* of partial differential equations and dynamical systems. Motivated by the facts that many of the celebrated (nonlinear) PDEs of mathematical physics can be viewed as Hamiltonian systems and that solutions of their linearized versions exhibit periodic or quasi-periodic solutions, Walter Craig (McMaster University, Canada) presents an overview of some of the techniques and results of KAM-like methods for analyzing analogous phenomena in solutions to nonlinear PDEs. Norman Dancer (University of Sydney, Australia) surveys a number of open questions for PDEs with small diffusion,  $-\varepsilon^2 \Delta u = f(u)$ , in various geometries: of interest are results on the number of solutions and their asymptotic shapes. The contribution by Christiane Rousseau (Université de Montréal, Canada), on normal forms for germs of analytic families of planar vector fields unfolding a generic saddle-node or resonant saddle, is concerned with the case in which the normal forms are not polynomial but analytic and for which the formal change of coordinates to normal form generically diverges.

Finally, Radu Saghin (University of Toronto, Canada) and Zhihong Xia (Northwestern University, USA) study generic properties of two particular classes of dynamical systems, namely symplectic diffeomorphisms and Hamiltonian systems.

Applications in biological sciences and in materials science are surveyed in five contributions. Two of these papers discuss biological invasions and disease spread: Julien Arino (University of Manitoba, Canada) and Pauline van den Driessche (University of Victoria, Canada) look at extensions of continuous time and discrete space metapopulation models addressing cross infection between several species and keeping track of the patches in which the species reside, while Stephen Gourley (University of Surrey, UK) and Jianhong Wu (York University, Canada) study nonlocal diffusive equations that arise in the modelling and analysis of long-term behaviors of biological and epidemiological systems where individuals move randomly and the feedback nonlinearity involves time lags.

Predator-prey and competition models are the subjects of the survey by Yihong Du (University of New England, Australia) and Junping Shi (College of William and Mary, USA) and the research paper by Gail Wolkowicz (McMaster University, Canada), respectively. The former presents recent results on diffusive predator-prey models in spatially heterogeneous environments and then examines the influence of a protection zone in a particular diffusive model. The paper by Wolkowicz studies the global dynamics of a Lotka-Volterra system of three species competition by using a model of a food web in a chemostat involving three species competing for a single (non-reproducing) nutrient, with one of the competitors also preying on one of the other two competitors.

Brian Sleeman (University of Leeds, UK) presents an illuminating description of how mathematical models may be formulated on the basis of the complex biochemical processes involved in the modelling of tumour angiogenesis (the formation of new blood vessels), and he describes various properties of solutions to such models (such as local existence and uniqueness, and the development of spikes).

A different field of applications (materials science) is reviewed in the survey by Peter Bates (Michigan State University, USA). In his discussion of nonlinear evolution equations arising in materials science, he describes various properties such as well-posedness, asymptotics, travelling waves or pulses. These systems represent lattice or nonlocal versions of the Allen-Cahn, Cahn-Hilliard, phase-field, or Klein-Gordon equations.

The other invited research papers are by Jifa Jiang (Tongji University, China) who analyzes the asymptotic behavior for systems that are comparable to quasi-monotone systems; by Tibor Krisztin (University of Szeged, Hungary), on  $C^1$ -smoothness of center manifolds for differential equations with state-dependent delay; and by Yingfei Yi (Georgia Institute of Technology, USA) and Xiang Zhang (Shanghai Jiaotong University, China) who present, among other things, a characterization of exact Poisson structures which are invariant under the flow of a class of completely integrable systems.

It has been the aim of the editors to create a proceedings volume that may serve as an important resource, both for new researchers and experts in this promising area, for many years to come. The editors wish to thank the Fields Institute for Research in Mathematical Science and its Editorial Board (chaired by Carl Riehm) for agreeing to include this set of papers in the Fields Institute Communications series.

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