Development of a Fast Vortex Method for Fluid Flow Simulation using Special-Purpose Computers

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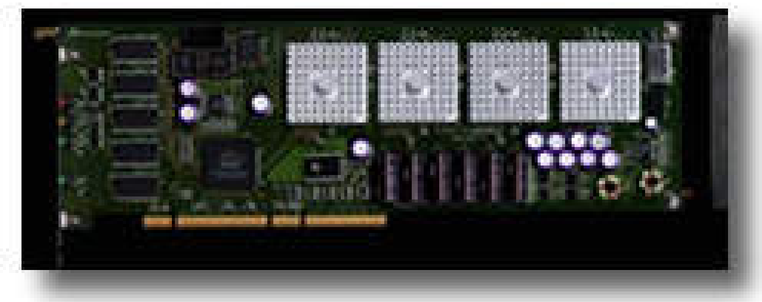


1
Introduction

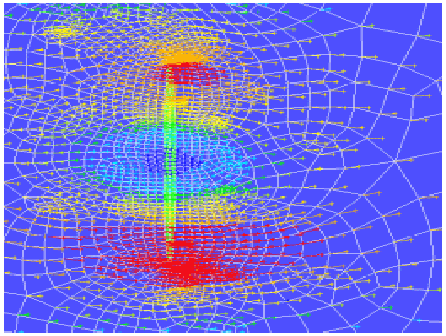
1.1 Background

- Vortex methods introduced in 1930s by Rosenhead
- Developed for complicated, unsteady and vortical flows
- N-Body algorithm introduced in 1950s
- Digital computers introduced in 1970s
- N-Body simulation is particle-interactions and calculation cost is $O(N^2)$
- Vortex method is one of the N-body algorithms

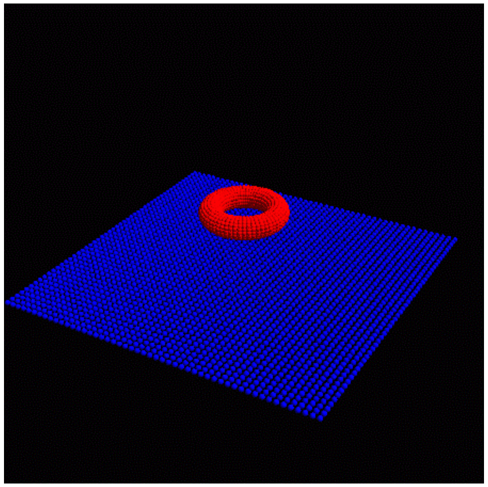
1.1 Background(Contd…)

- Advantages of Vortex Method (VM)
  - Lagrangian based CFD method
  - Calculates only regions of non-zero vorticity
  - Can be applied to high Reynolds number flows in complex geometry
  - Can be solved convection in straightforward
- Disadvantages of VM
  - High computation cost
  - Descretization error
  - Diffusion error

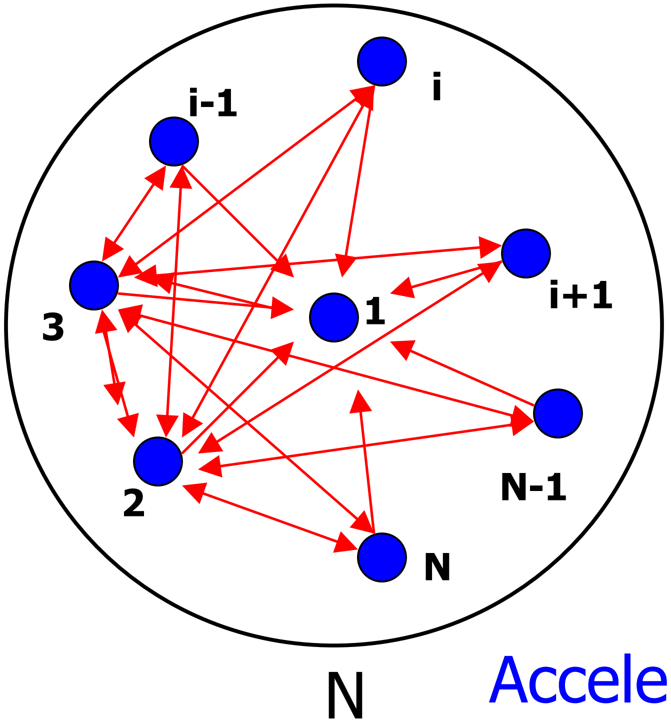
ANSYS Inc.



Asakura, 2002



1.2 The need for acceleration techniques



N² Calculation

Accelerating N-body calculation

Hardware: parallel calculation

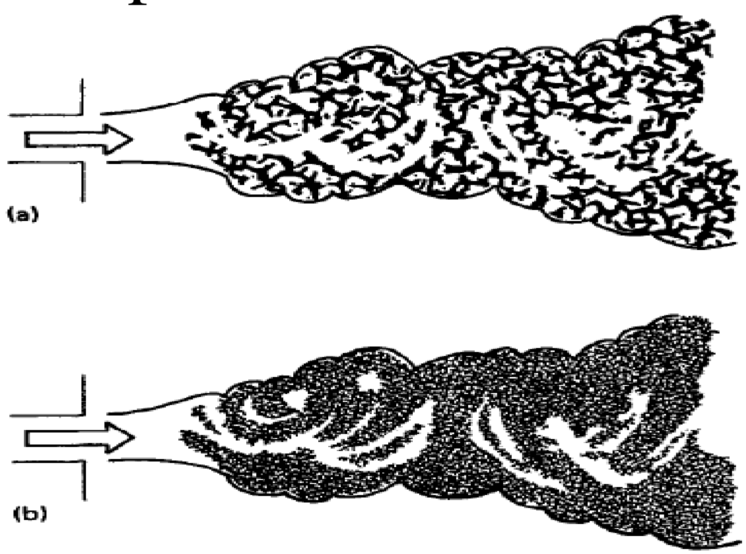
Software: Tree code, FMM, P^2M^2 Pseudo-particle multipole method etc.

1.3 Motivation

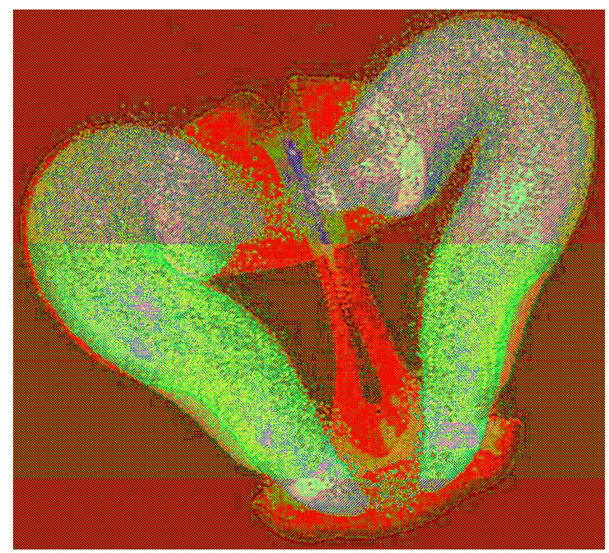
- Two ways to reduce calculation cost
  - Fast algorithms (Cheng, 1999)
  - Special-purpose computers (Susukita, 2003; Narumi, 2006)
- Fast algorithm has high proportionality cost at higher accuracy (Greengard, 1987)
- Special-purpose computers have been developed to accelerate MD simulations (Narumi, 1997)
  - It can be applied to accelerate VM calculation
  - Fast algorithms can be implemented
  - It does not support for Fast Poisson solver
- Recently GPGPU has been used to accelerate VM calculation (Stock, 2008)

1.3 Motivation(Contd…)

- To calculate for high Reynolds number turbulent flows which required high performance computational resources



Tennekes and Lumely, A first course In turbulence



Kida, 1994

- The collision of vortex rings contain millions of particles result in a highly turbulent state

1.4 Previous Studies

- Vortex rings have been studied in the broader arena of vortex interaction (Shariff, 1992)
- Large N is necessary to capture the essential characteristics of vortex rings collisions (Winckelmans, 1993)
- High Reynolds number is necessary to generate a secondary vortex rings (Mammetti, 1999)
- Computational resources are essential for longer calculation and to produce the fast mechanism of energy transfer (Chatelain, 2003)
- Fast Poisson solvers are still faster compared with VIC method (Cottet, 2002)
- Fast algorithm is successfully implemented on special-purpose computers for astrophysical problems (Makino, 1991, Kawai, 2004)

1.5 Purpose of the present study

- To accelerate the high Reynolds number VM calculation without loss of numerical accuracy
- To develop a fast vortex method using special-purpose computers
- To solve the three critical issues
  - The efficient calculation of Biot-Savart law and stretching term
  - An optimized function table
  - Round-off error caused by the partially single precision of MDGRAPE
  - Special treatments for cross product calculation
- To implement fast algorithms on special-purpose computers for further acceleration
- Comparative study to validate this scheme

2
Numerical Methods

2.2 Vortex Methods

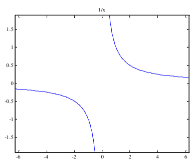
- Lagrangian methods used to simulate unsteady, convection-dominated problems
- It has a difficulty in achieving higher order spatial accuracy compared to Eulerian methods
- It is required to consider an accurate viscous diffusion scheme

2.2.1 Formulation of 3D VM

- Vorticity transport: $$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \boldsymbol{\omega} \qquad (2.1)$$
- Biot-Savart Law: $$\mathbf{u}(\mathbf{x}) = -\frac{1}{4\pi} \int \frac{(\mathbf{x}-\mathbf{x}') \times \boldsymbol{\omega}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} dV(\mathbf{x}') \qquad (2.2)$$
- Discretized form: $$\mathbf{u}_i = -\frac{1}{4\pi} \sum_{j=1}^{N} \frac{\mathbf{r}_{ij}^2 + (5/2)\sigma_j^2}{(\mathbf{r}_{ij}^2 + \sigma_j^2)^{5/2}} \mathbf{r}_{ij} \times \boldsymbol{\gamma}_j \qquad (2.3)$$
- Stretching term: $$\frac{d\gamma_i}{dt} = \frac{1}{4\pi} \sum_{j=1}^{N} \left\{ -\frac{\mathbf{r}_{ij}^2 + (5/2)\sigma_j^2}{(\mathbf{r}_{ij}^2 + \sigma_j^2)^{5/2}} \boldsymbol{\gamma}_i \times \boldsymbol{\gamma}_j + 3\frac{\mathbf{r}_{ij}^2 + (7/2)\sigma_j^2}{(\mathbf{r}_{ij}^2 + \sigma_j^2)^{7/2}} (\boldsymbol{\gamma}_i \cdot \mathbf{r}_{ij})(\mathbf{r}_{ij} \times \boldsymbol{\gamma}_j) \right\} \qquad (2.6)$$

Core Spreading Method

- Viscous Diffusion: $$\frac{d\omega_i}{dt} = \nu\nabla^2\omega_i \qquad (2.7)$$
- Green's Function Solution: $$\boldsymbol{\omega}_i = \frac{\boldsymbol{\gamma}_j}{(4\pi\nu t)^{d/2}}\exp\left(-\frac{\left|\mathbf{x}_j-\mathbf{x}_i\right|^2}{4\nu t}\right) \qquad (2.8)$$



- Vorticity at arbitrary point: $$\boldsymbol{\omega}_i = \sum_j \gamma_j \zeta\left(\left|\mathbf{x}_j-\mathbf{x}_i\right|\right) \qquad (2.9)$$

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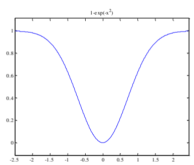

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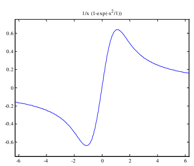
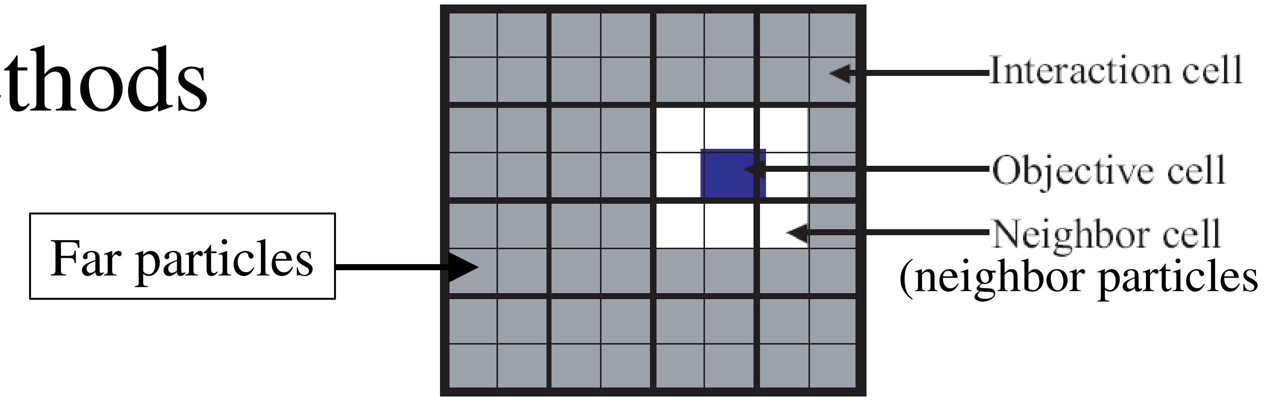
- Cutoff Function: $$\zeta = \frac{1}{(2\pi\sigma^2)^{d/2}}\exp\left(-\frac{\left|\mathbf{x}_j-\mathbf{x}_i\right|^2}{2\sigma^2}\right) \qquad (2.10)$$



- Core expansion rate: $$\sigma^2 = 2\nu t \qquad (2.11)$$
- Position update: $$\frac{d\mathbf{x}_i}{dt} = \mathbf{u}_i \qquad (2.12)$$

2.3 Fast Methods



- Tree Algorithm (Barnes and Hut, 1986)
  - Hierarchical data structure
  - Calculation cost O(N logN)
  - Can be implemented on special-purpose computers
- Fast Multipole Method (Greengard and Rokhlin, 1987)
  - All particles are uniformly distributed in a unit cube
  - Far particles calculated as a multipole expansion
  - Neighbor particles calculated in a direct summation
  - Calculation cost is proportional to O(N)
- Other fast methods
  - Anderson's method (Anderson, 1992)
  - Pseudo-particle multipole method (Makino, 1999)

2.3 Fast Methods(Contd…)

- FMM has been used in my calculation
- Biot-Savart equation has been derived as

$$\mathbf{u}_i \approx \frac{1}{4\pi}\sum_{n=0}^{p}\sum_{m=-n}^{n}\left\{\sum_{j=1}^{N}\gamma_j M_j\right\}\times\nabla S_i \qquad (2.13)$$

$$\mathbf{u}_i \approx \frac{1}{4\pi}\sum_{n=0}^{p}\sum_{m=-n}^{n}\left\{\sum_{j=1}^{N}\gamma_j L_j\right\}\times\nabla R_i \qquad (2.14)$$

- Stretching term derived as

$$\frac{D\gamma_i}{Dt} \approx \frac{1}{4\pi}\sum_{n=0}^{p}\sum_{m=-n}^{n}\left\{\sum_{j=1}^{N}\gamma_j \times\nabla M_j\right\}(\gamma_i \cdot\nabla S_i) \qquad (2.15)$$

$$\frac{D\gamma_i}{Dt} \approx \frac{1}{4\pi}\sum_{n=0}^{p}\sum_{m=-n}^{n}\left\{\sum_{j=1}^{N}\gamma_j \times\nabla L_j\right\}(\gamma_i \cdot\nabla R_i) \qquad (2.16)$$

Here p is order of moment and L, M, R, S are denoted to simplify the equations.

Summary

- A fast vortex method has been formulated
- 3D core spreading method is explained
- Different fast methods are reviewed briefly

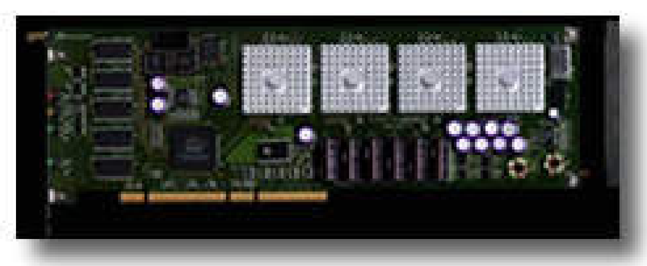
4
Fast Vortex Method Calculation using a Special-purpose Computer

4.1 Introduction

- A mathematical formulation will be developed for VM calculation using MDGRAPE-2
- The efficient calculation of Biot-Savart law and stretching term will be performed
- An optimum range of a function table is determined
- The cross-product calculation will be demonstrated
- The accuracy will be evaluated by calculating the impingement of two identical vortex rings
- The calculated results is compared with and without the use of MDGRAPE-2 and with referenced work carefully

Special-purpose Computer: MDGRAPE-2 (Narumi, 1997)

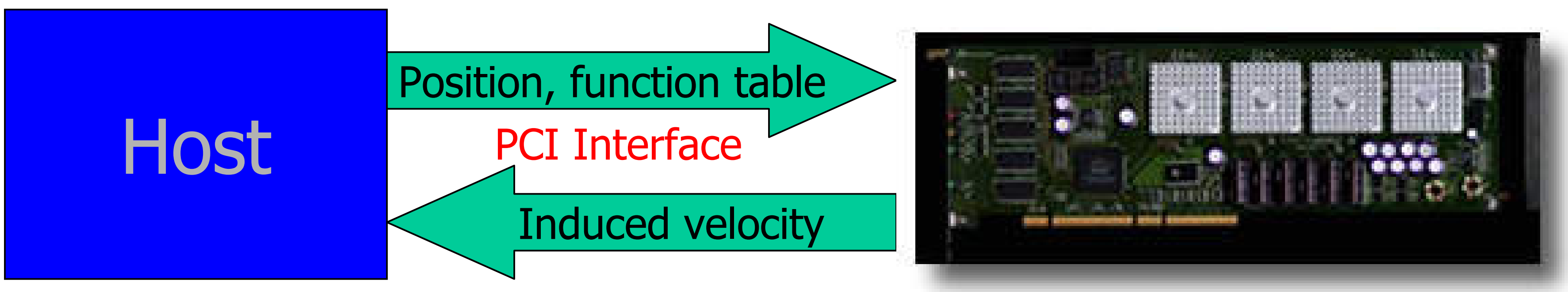
- PCI-board for accelerating Molecular Dynamics Simulation
- Reduced computation cost significantly for N^2 calculation
- Particle Memory: 5.0×10^5 (20MB)
- Calculation speed: 64 Gflops
- Speed up 10-1000 times faster
- Compatible for FORTRAN and C programming Languages



Host Machine and MDGRAPE-2

General-purpose
Time Integration, Vorticity, etc..

Special-purpose
Induced Velocity, Stretching Term



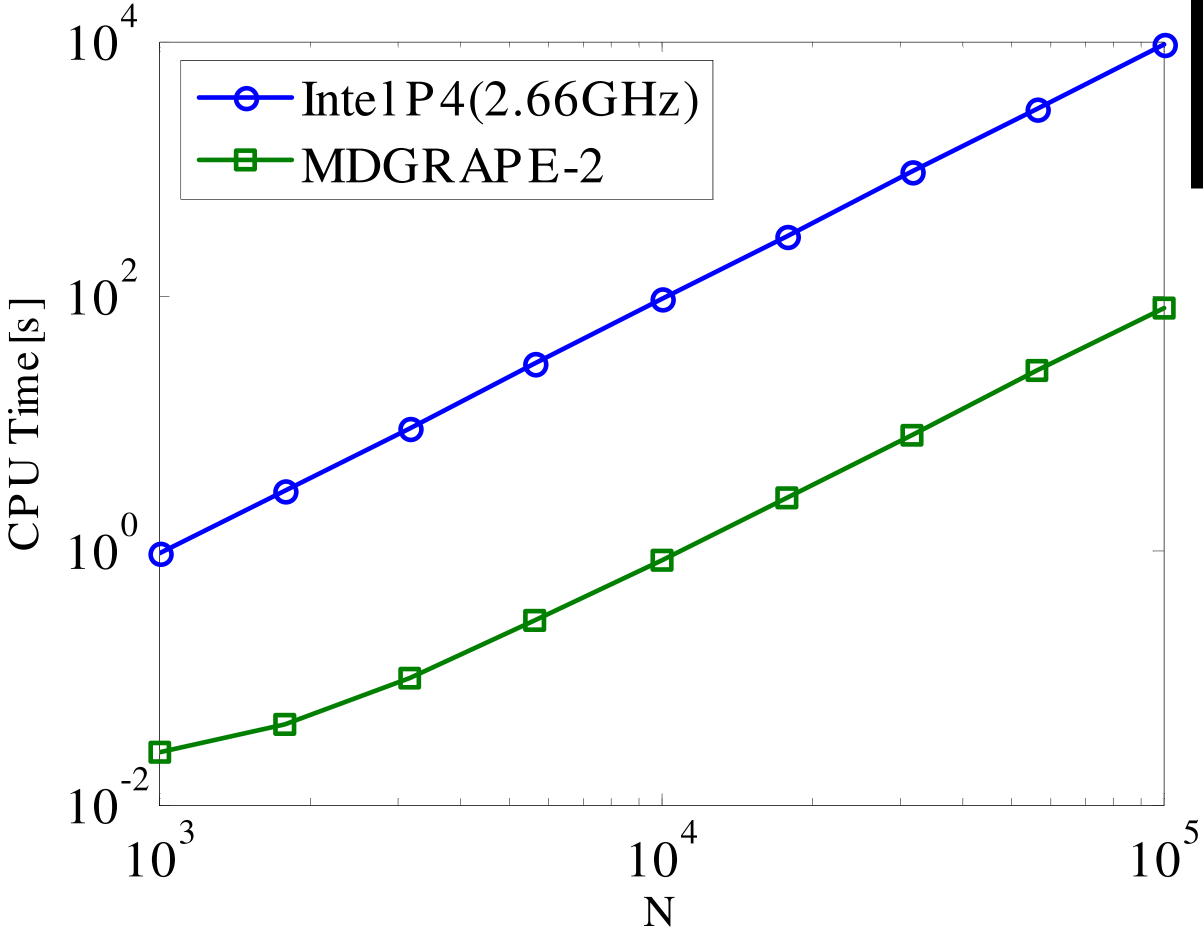
MDGRAPE-2

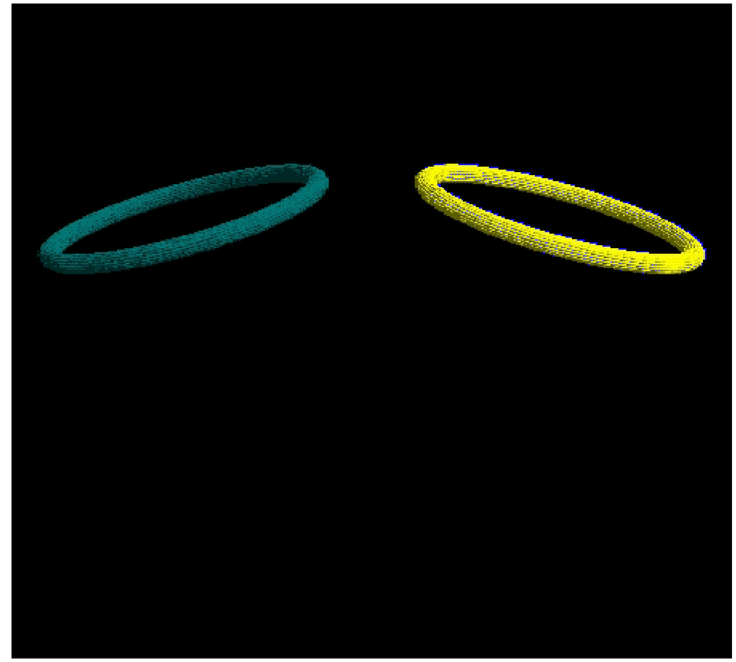
O(N) O(N) $O(N^2)$

Hosts for performance measurement

Special-purpose	Host CPU	Cache	Memory	OS	Compiler
MDGRAPE-2	Intel P4 2.66GHz (1CPU 1 Core)	512KB	1GB (2 GB Swap Memory)	Linux 8.0 Kernel 2.4.18-14	ifort
MDGRAPE-3	Xeon 5160 3.0GHz (1CPU 2 Core)	4096KB	32GB (0 GB Swap Memory)	Cent 4.3 (Final) Kernel 2.6.9-34_ELsmp	ifort

Performance





N=6x10^4

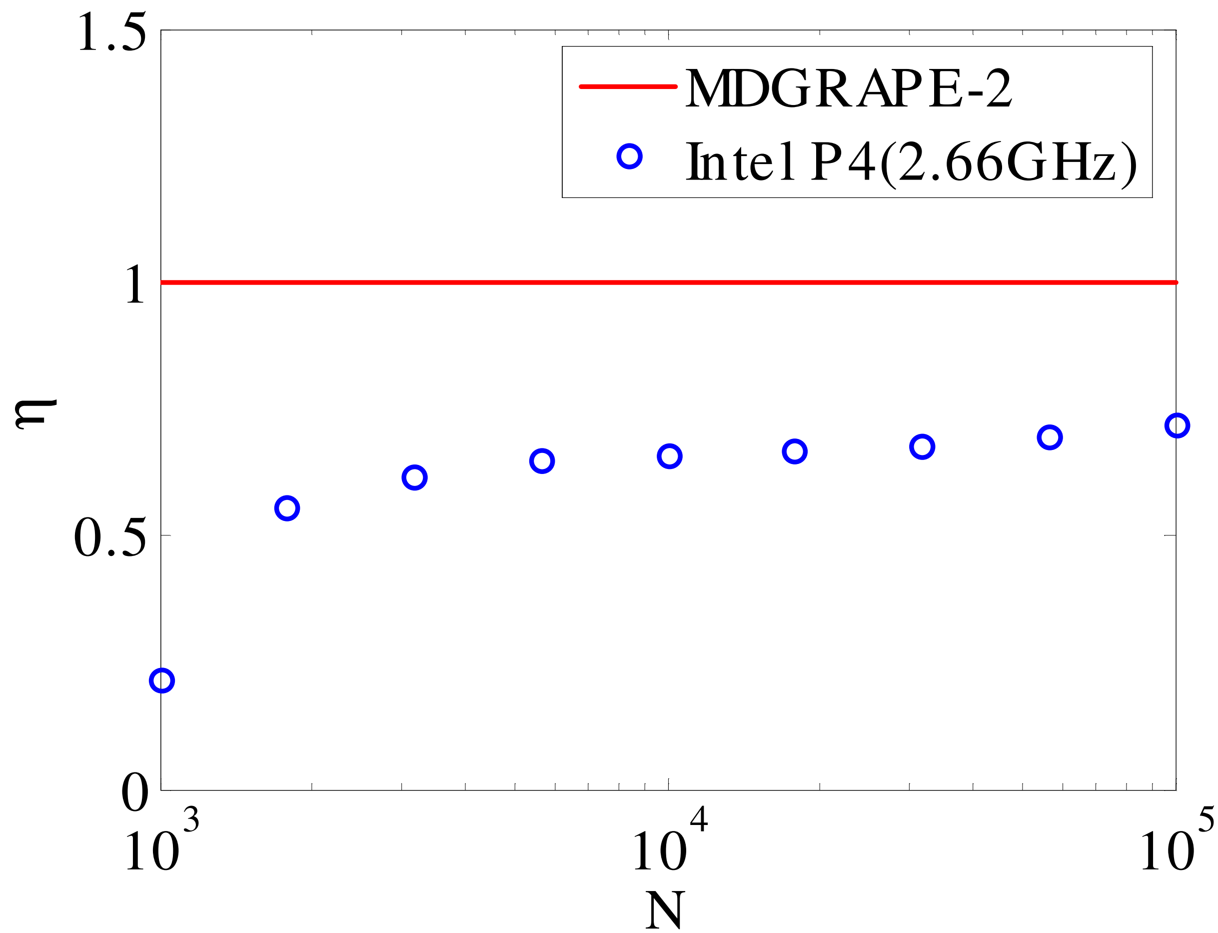
100 times at N~10^5

Efficiency of VM calculation

$$N_{APPL} = \frac{nmd(xmd + ymd + zmd)}{CPUtime(\mathrm{sec}/\,step)}; \quad N_{GRAPE} = 5.5\times 10^{8}$$

Efficiency:

$$\eta = \frac{N_{APPL}}{N_{GRAPE}}$$



Difficulties with MDGRAPE-2 for VM

- Not designed for vector product calculation
- Requires optimum generation of a function table
- Calculates partly with single precision

Special treatments are required to solve these problems

4.2 Mathematical Formulations

Coulomb Potential: $$\Phi_i = \sum_{j=1}^{N} b_{ij} g(w) \mathbf{r}_{ij} = \sum_{j=1}^{N} b_{ij} g\left(a_{ij}\left(\left|\mathbf{r}_{ij}\right|^2 + \varepsilon_{ij}^2\right)\right) \qquad (4.1)$$

Coulomb Force: $$\mathbf{f}_i = \sum_{j=1}^{N} b_{ij} g(w) \mathbf{r}_{ij} = \sum_{j=1}^{N} b_{ij} g\left(a_{ij}\left(\left|\mathbf{r}_{ij}\right|^2 + \varepsilon_{ij}^2\right)\right)\mathbf{r}_{ij} \qquad (4.2)$$

Vortex Method (Induced velocity)

$$\mathbf{u}_i = \sum_{j=1}^{N} \frac{\mathbf{r}_{ij}^2 + (5/2)\sigma_j^2}{4\pi\left(\mathbf{r}_{ij}^2 + \sigma_j^2\right)^{5/2}} \boxed{\mathbf{r}_{ij} \times \boldsymbol{\gamma}_j}$$

$$\mathbf{u}_i = \sum_{j=1}^{N} B_j g\left(A_j\left(\left|\mathbf{r}_{ij}\right|^2 + \varepsilon_{ij}^2\right)\right)\mathbf{r}_{ij} \qquad (4.3)$$

4.2 Mathematical Formulations (contd.)

- Input

$$\mathbf{r}_{ij} = \left(x_{ij}, y_{ij}, z_{ij}\right) \qquad \boldsymbol{\gamma}_j = \left(\gamma_j^x, \gamma_j^y, \gamma_j^z\right) \qquad (4.4)$$

- Vector product

$$\sum_j \mathbf{r}_{ij} \times \boldsymbol{\gamma}_j = \sum_j \left(y_{ij}\gamma_j^z - z_{ij}\gamma_j^y, z_{ij}\gamma_j^x - x_{ij}\gamma_j^z, x_{ij}\gamma_j^y - y_{ij}\gamma_j^x\right) \quad (4.5)$$

- 3 x Scalar product

$$\sum_j \mathbf{r}_{ij} \cdot \gamma_j^x = \sum_j \left(x_{ij}\gamma_j^x, y_{ij}\gamma_j^x, z_{ij}\gamma_j^x\right) \to \sum_j \left(0, y_{ij}\gamma_j^x, z_{ij}\gamma_j^x\right) \quad (4.6)$$

$$\sum_j \mathbf{r}_{ij} \cdot \gamma_j^y = \sum_j \left(x_{ij}\gamma_j^y, y_{ij}\gamma_j^y, z_{ij}\gamma_j^y\right) \to \sum_j \left(x_{ij}\gamma_j^y, 0, z_{ij}\gamma_j^y\right) \quad (4.7)$$

$$\sum_j \mathbf{r}_{ij} \cdot \gamma_j^z = \sum_j \left(x_{ij}\gamma_j^z, y_{ij}\gamma_j^z, z_{ij}\gamma_j^z\right) \to \sum_j \left(x_{ij}\gamma_j^z, y_{ij}\gamma_j^z, 0\right) \quad (4.8)$$

Mathematical Formulations(contd.)

$$\mathbf{u}_i = -\frac{1}{4\pi}\sum_j \frac{1}{\sigma_j^3} g_1(w)(\mathbf{r}_{ij} \times \gamma_j) \qquad (4.9)$$

$$\mathbf{stx} = -\frac{1}{4\pi}\sum_j g_1(w)\left(\gamma_i^y\gamma_j^z - \gamma_i^z\gamma_j^y, \gamma_i^z\gamma_j^x - \gamma_i^x\gamma_j^z, \gamma_i^x\gamma_j^y - \gamma_i^y\gamma_j^x\right)\frac{1}{\sigma_j^3} \qquad (4.11)$$

$$\mathbf{tx} = \frac{3}{4\pi}\sum_j g_2(w)(\gamma_i \cdot \mathbf{r}_{ij})\left(y_{ij}\gamma_j^z - z_{ij}\gamma_j^y, z_{ij}\gamma_j^x - x_{ij}\gamma_j^z, x_{ij}\gamma_j^y - y_{ij}\gamma_j^x\right)\frac{1}{\sigma_j^5} \qquad (4.12)$$

$$\mathbf{I}_i = (\gamma_i \cdot \mathbf{r}_i)\mathbf{S} - \left(\gamma_i^x\mathbf{T1} + \gamma_i^y\mathbf{T2} + \gamma_i^z\mathbf{T3}\right) \qquad (4.14)$$

Here,

$$w = \left(|\mathbf{r}_{ij}|/\sigma_j\right)^2; \quad |\mathbf{r}_{ij}| = |\mathbf{r}_i - \mathbf{r}_j|;$$

Function Table and Coefficients

$g(w)$	A_j	B_j	ε_{ij}
$g1(w) = \frac{w + 5/2}{(w+1)^{5/2}}$	$\frac{1}{\sigma_j^2}$	$\frac{\boldsymbol{\gamma}_j}{\sigma_j^3}$	0
$g2(w) = \frac{w + 7/2}{(w+1)^{7/2}}$	$\frac{1}{\sigma_j^2}$	$\frac{\boldsymbol{\gamma}_j}{\sigma_j^5}$	0

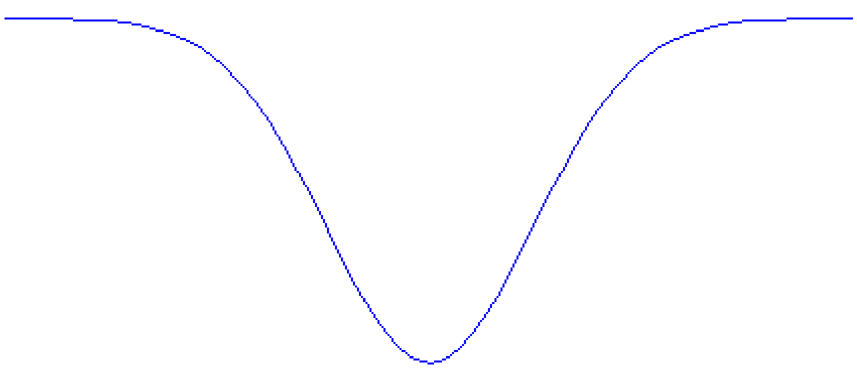
Function Table

Function g(w):

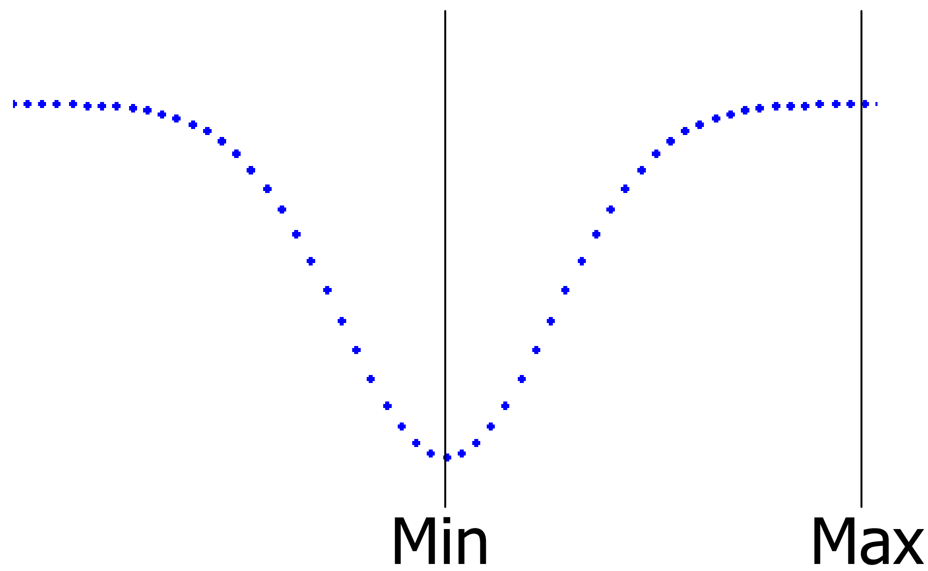
$$g(w): \min \le w \le \max \quad \text{(Domain of function)}$$

$$\frac{w_{\max}}{w_{\min}} \le 2^{32}$$

g(w)

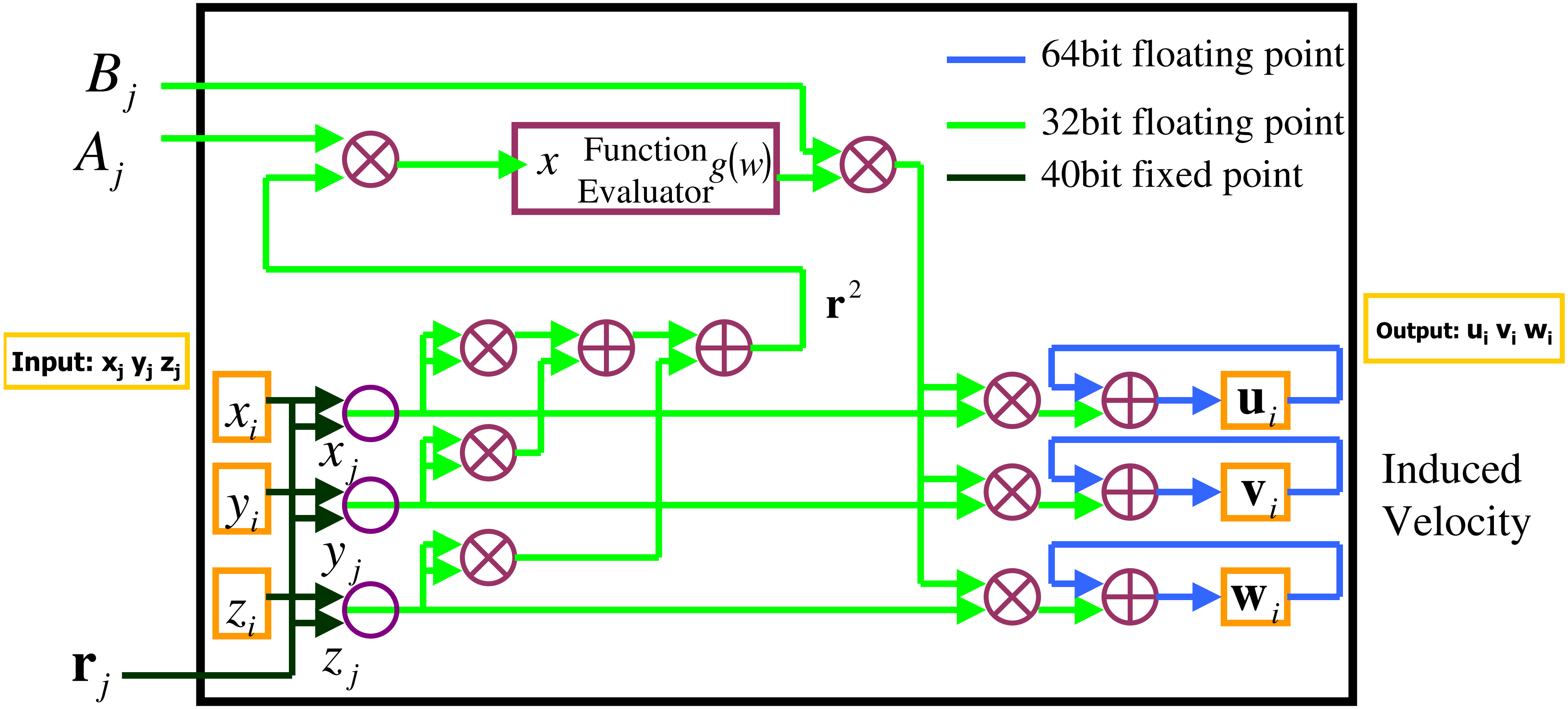


Function Table



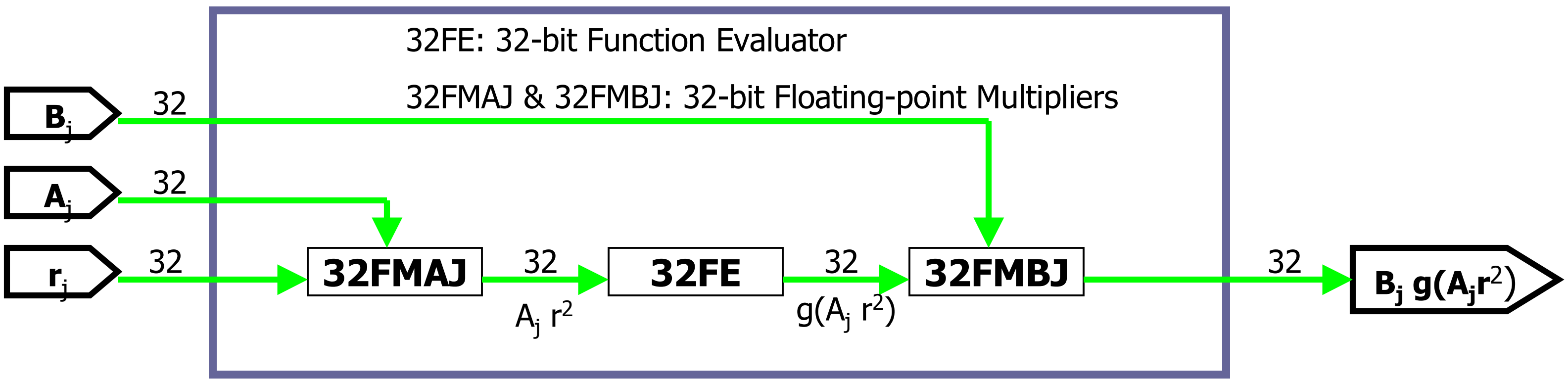
Block Diagram of MDGRAPE-2 pipeline

Pairwise force: $\vec{f}_{i,j} = b_{ij}\, g\left(a_{ij} r_{ij}^2\right) \vec{r}_{ij}$ (3.1)



Ref: Tetsu Narumi, 1997, PhD Thesis, Tokyo University

Function Evaluator



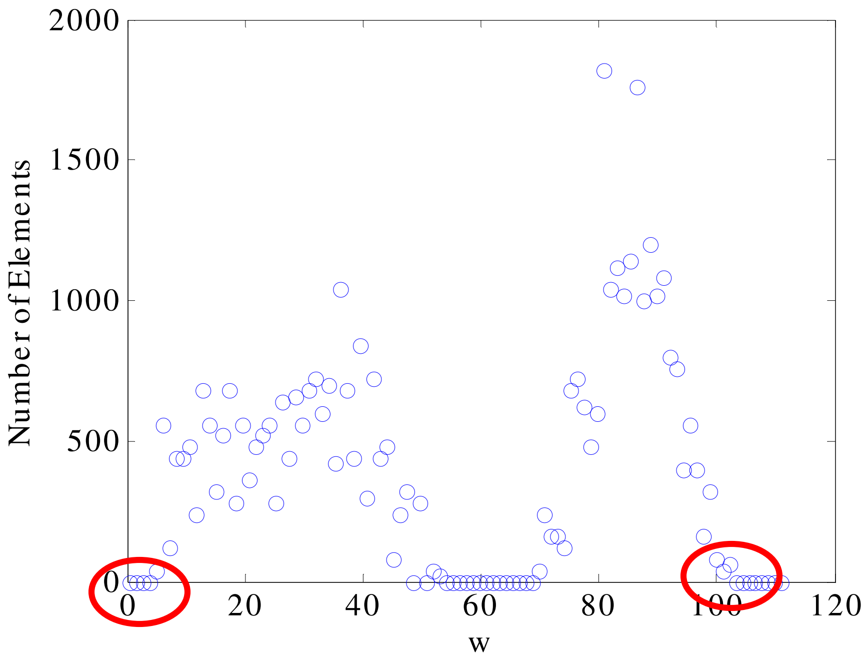
Approximation:

$$g(w) = c_0 + w(c_1 + w(c_2 + w(c_3 + wc_4)))$$

- 32 bit floating-point calculation
- Relative accuracy: 10^{-7}

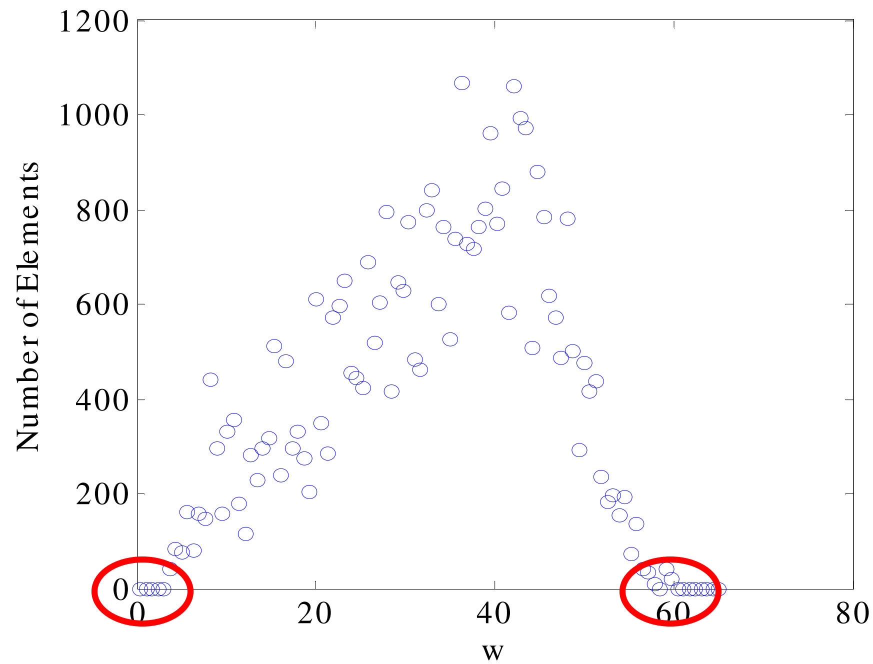
4.3 Typical distribution of vortex elements

$t\Gamma / R^2 = 1$



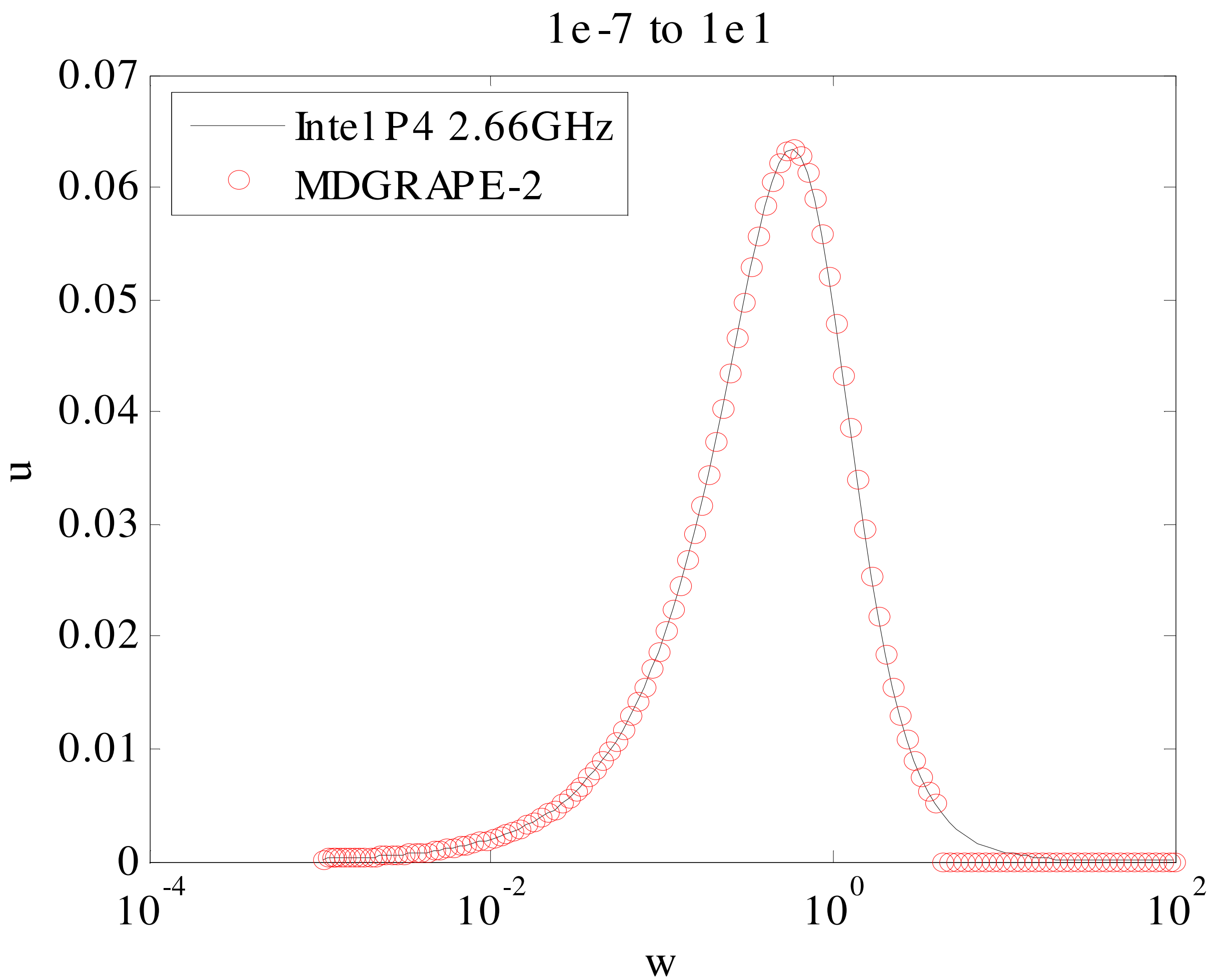
Min:3.9; Max: 103.2

$t\Gamma / R^2 = 100$



Min:2.94; Max: 60.37

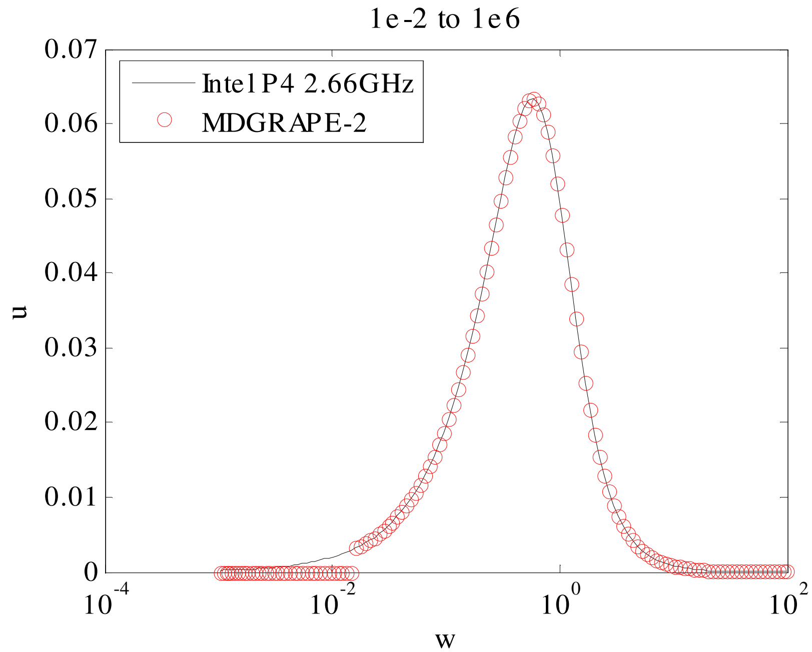
Range of a Function Table

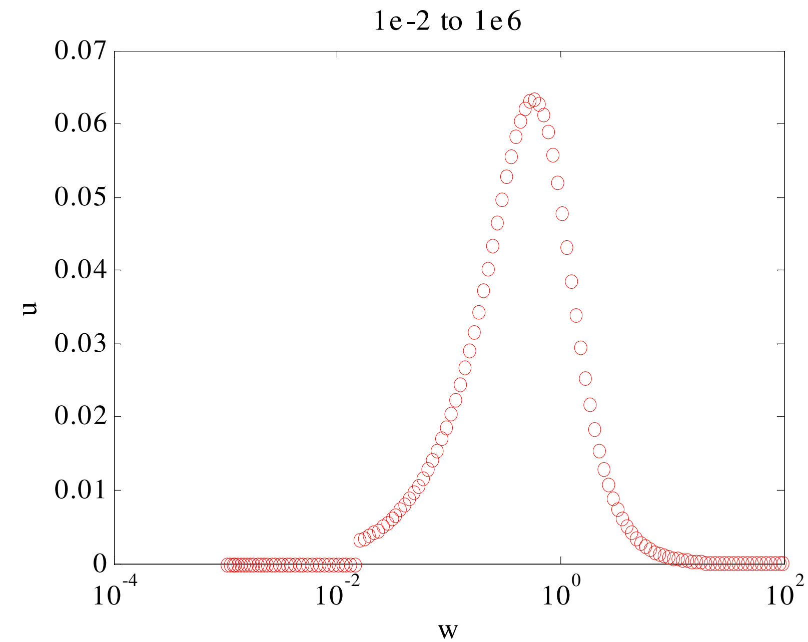


tested table ranges

min	max
10^{-2}	10^{6}
10^{-3}	10^{5}
10^{-4}	10^{4}
10^{-5}	10^{3}
10^{-6}	10^{2}
10^{-7}	10^{1}

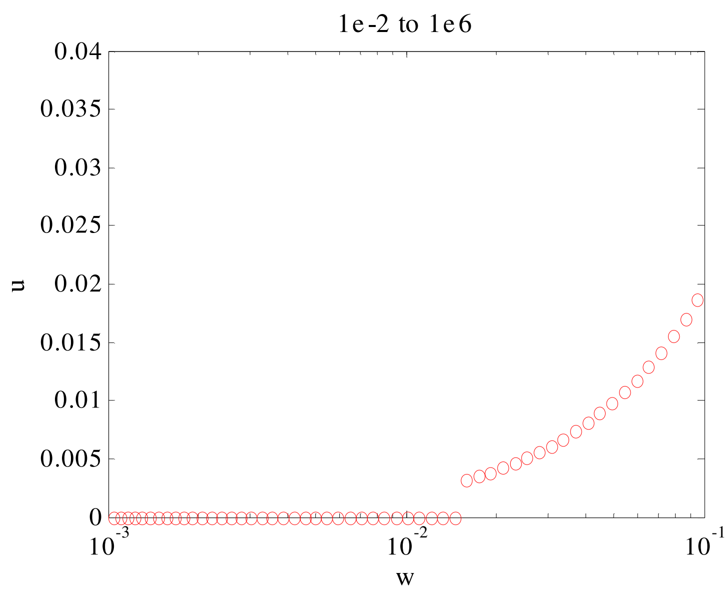
4.3.4 Optimum Range of a Function Table

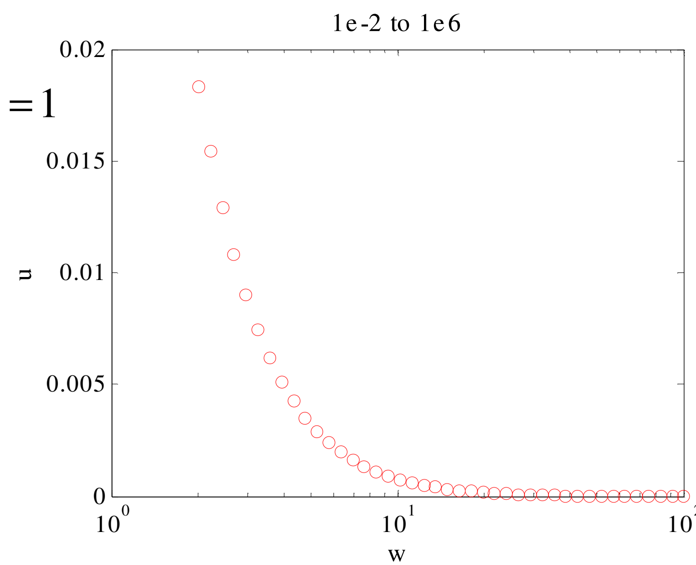




$2^{-12} < w < 2^{20}$

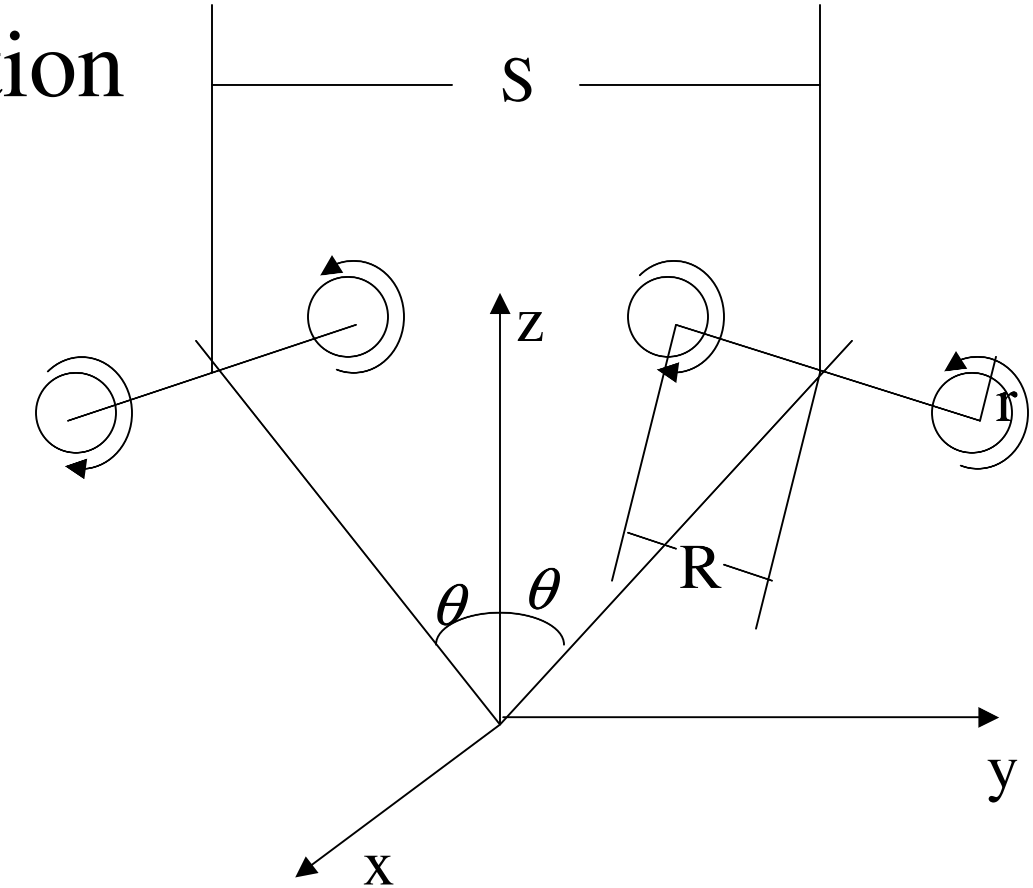
$$w = \left(\mathbf{r}_{ij}/\sigma_j\right)^2; \quad here \quad \sigma = 1$$





4.5 Application

Calculation Condition



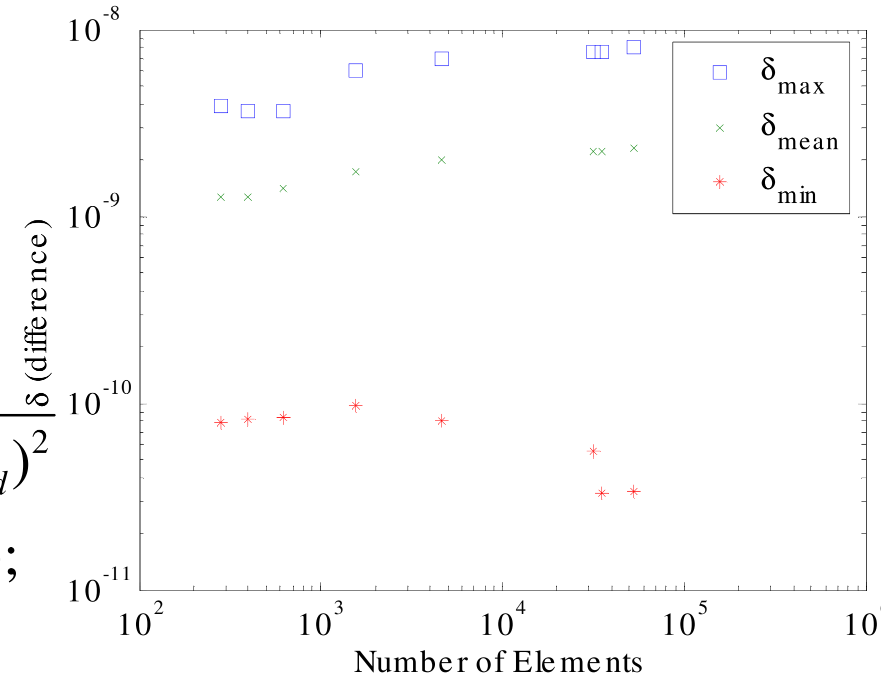
R = Radius of ring
r = Radius of cross-section
S = Distance between the center of two rings
θ = Inclined angle of a ring

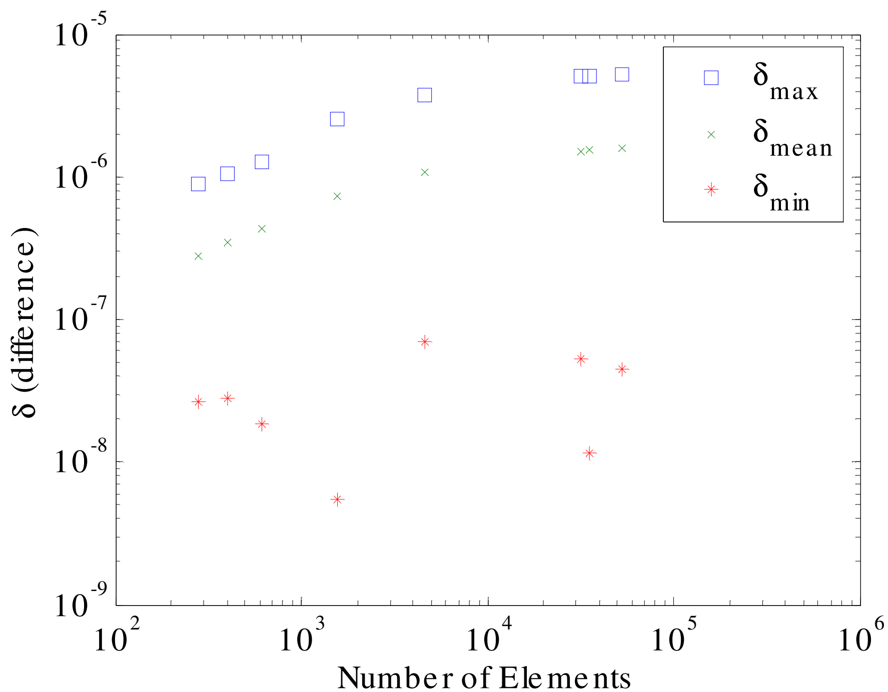
Position	R	r	σ	Γ_0	N	Re_Γ	S	θ
Inclined	1	0.05	0.065	1	$2\times502\times61$	400	2.7	15^0

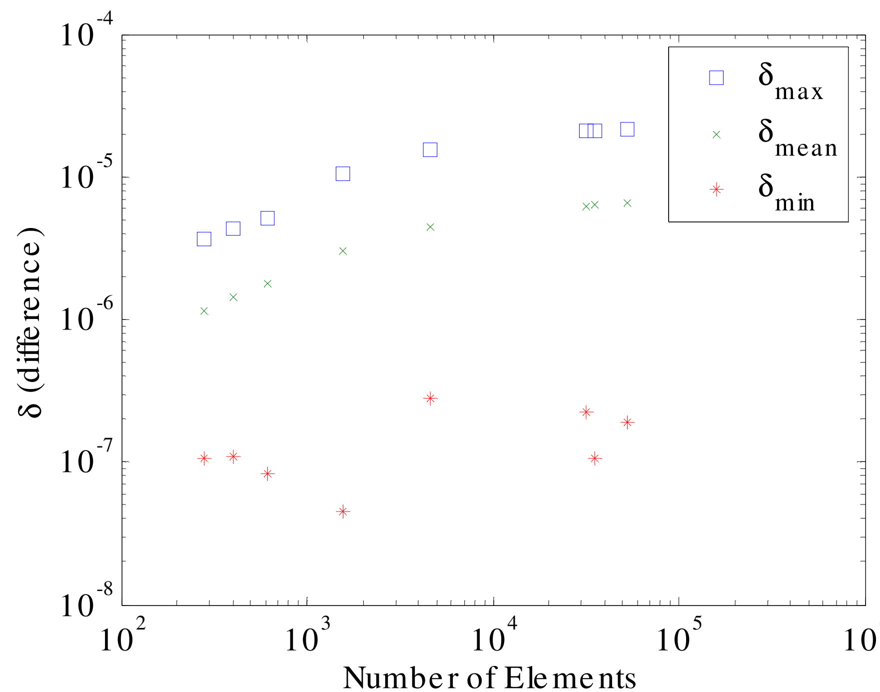
Convection error

$$\delta = \sqrt{(x_{host} - x_{md})^2 + (y_{host} - y_{md})^2 + (z_{host} - z_{md})^2}$$

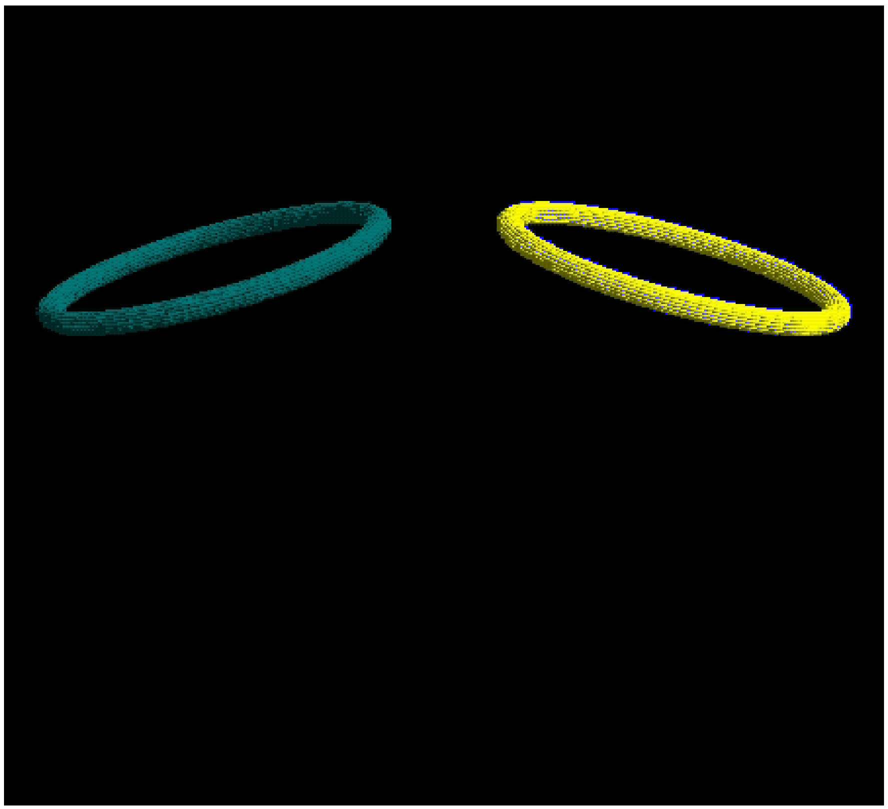
$$\delta_{\min} = \min(\delta); \delta_{mean} = mean(\delta); \delta_{\max} = \max(\delta);$$

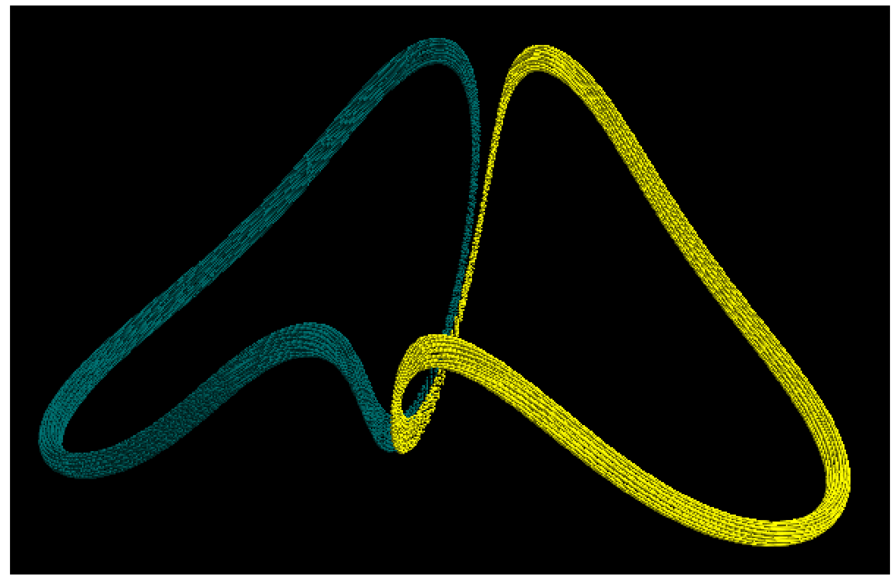


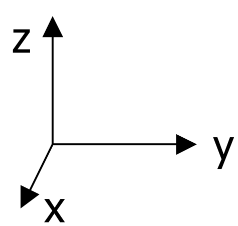




Snapshots of vortex elements







Ring radius R=1

Cross-section radius r=0.05

Reynolds number Re=400

Core radius = 0.065, Circulation of ring = 1

Total number of elements = 2x502x61

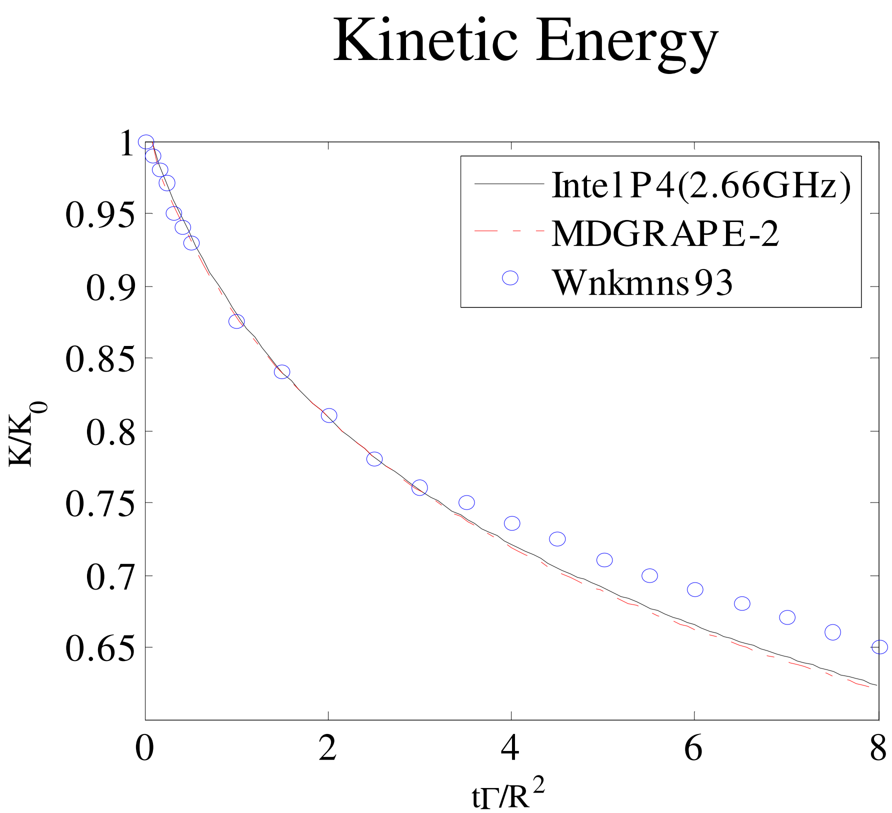
Particles are evenly distributed

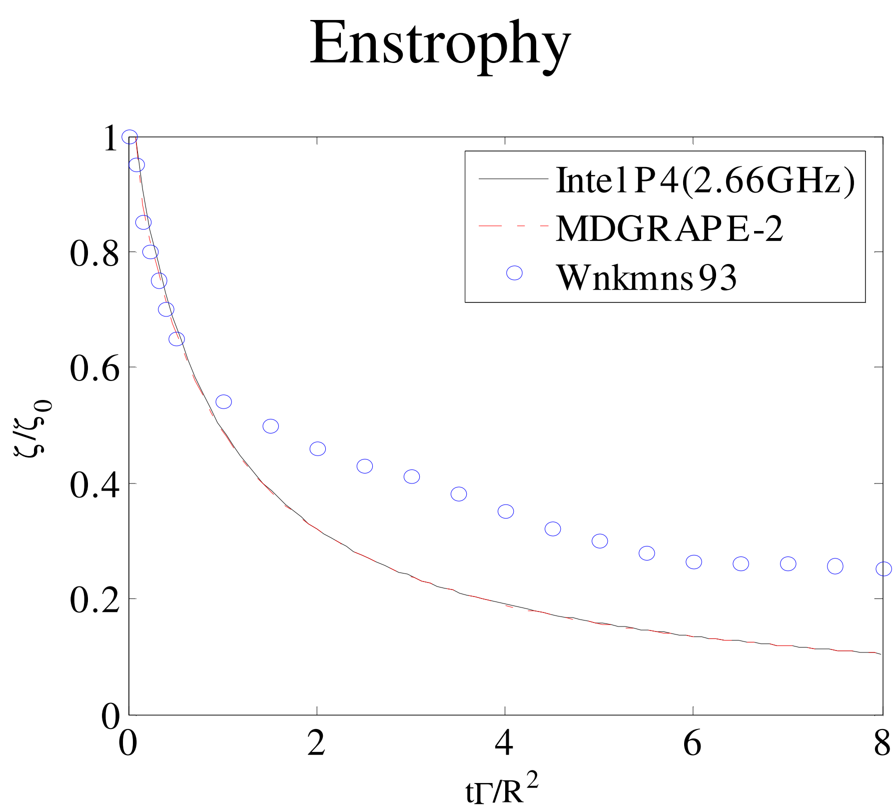
Initial distance between two rings = 2.7

Inclined angle = 15^0

Kinetic Energy & Enstrophy

Kinetic energy: Eq. (4.16) and Enstrophy: Eq. (4.17)





Wnkmns93: G. S. Winckelmans and Leonard, J. Comp. Phys, 109, 247-273(1993)

Summary

- A mathematical formulation has been developed using MDGRAPE-2
- A rigorous assessment of this hardware has been made for a pair of impinging vortex rings
- Computational domain has been investigated that determines optimal range of a function table
- The global kinetic energy and enstrophy has been evaluated to address the numerical accuracy
- The results have good agreement when compared with the host calculation and referenced work

5
The Study of Colliding Vortex Rings using a Special-purpose Computer and FMM

5.1 Introduction

- Simultaneous use of the FMM with MDGRAPE-3
- To investigate the possibility of further accelerations
- The various forms of FMM are investigated
- The accuracy is achieved by simulating the impingement of two identical inclined vortex rings
- The effect of temporal and spatial resolutions will be investigated
- The reconnection of vortex rings is observed

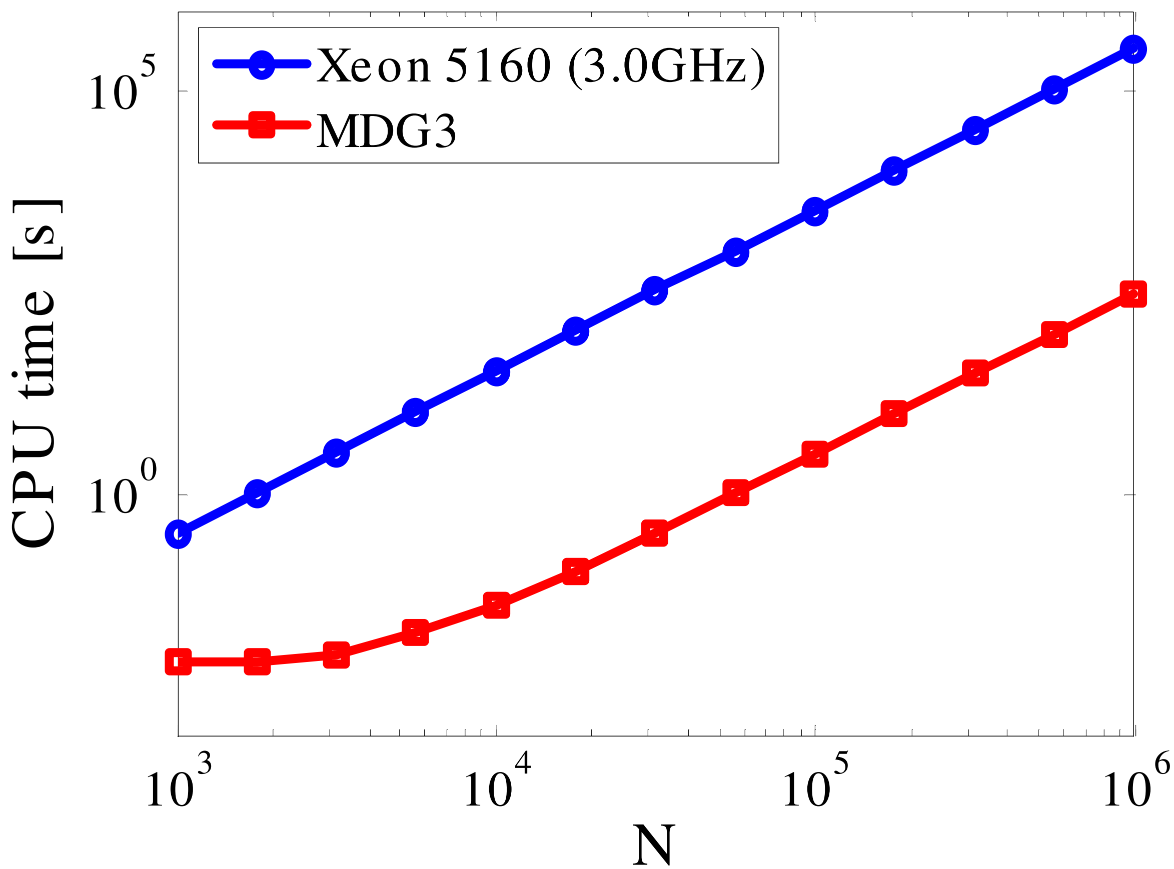
Special-purpose Computer: MDGRAPE-3

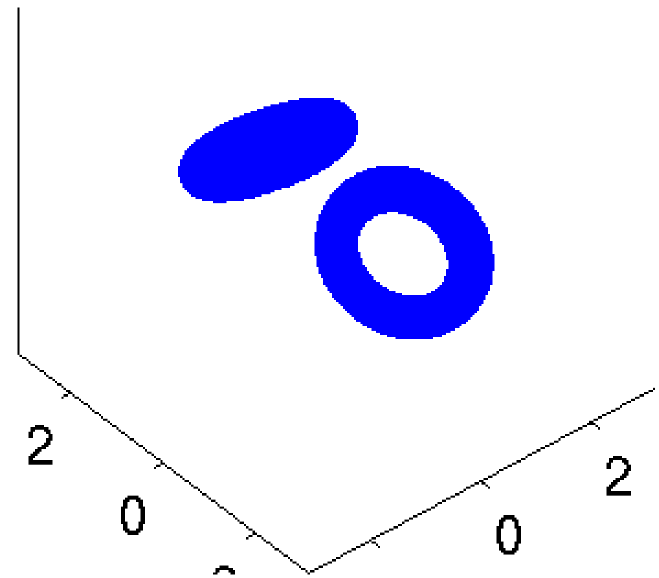
（Taiji, 2003; Narumi, 2006 ）

- Petaflops special-purpose computer and the successor of MDGRAPE-2
- One small board consists of 2 chips
- One MDGRAPE-3 chip combined with 20 parallel pipelines
- Calculation speed of one small board is 330GFlops/250MHz
- 12.5 times faster than MDGRAPE-2
- Total memory/chip: 9Mbits

MDGRAPE-3 Board

Performance





N=10^6

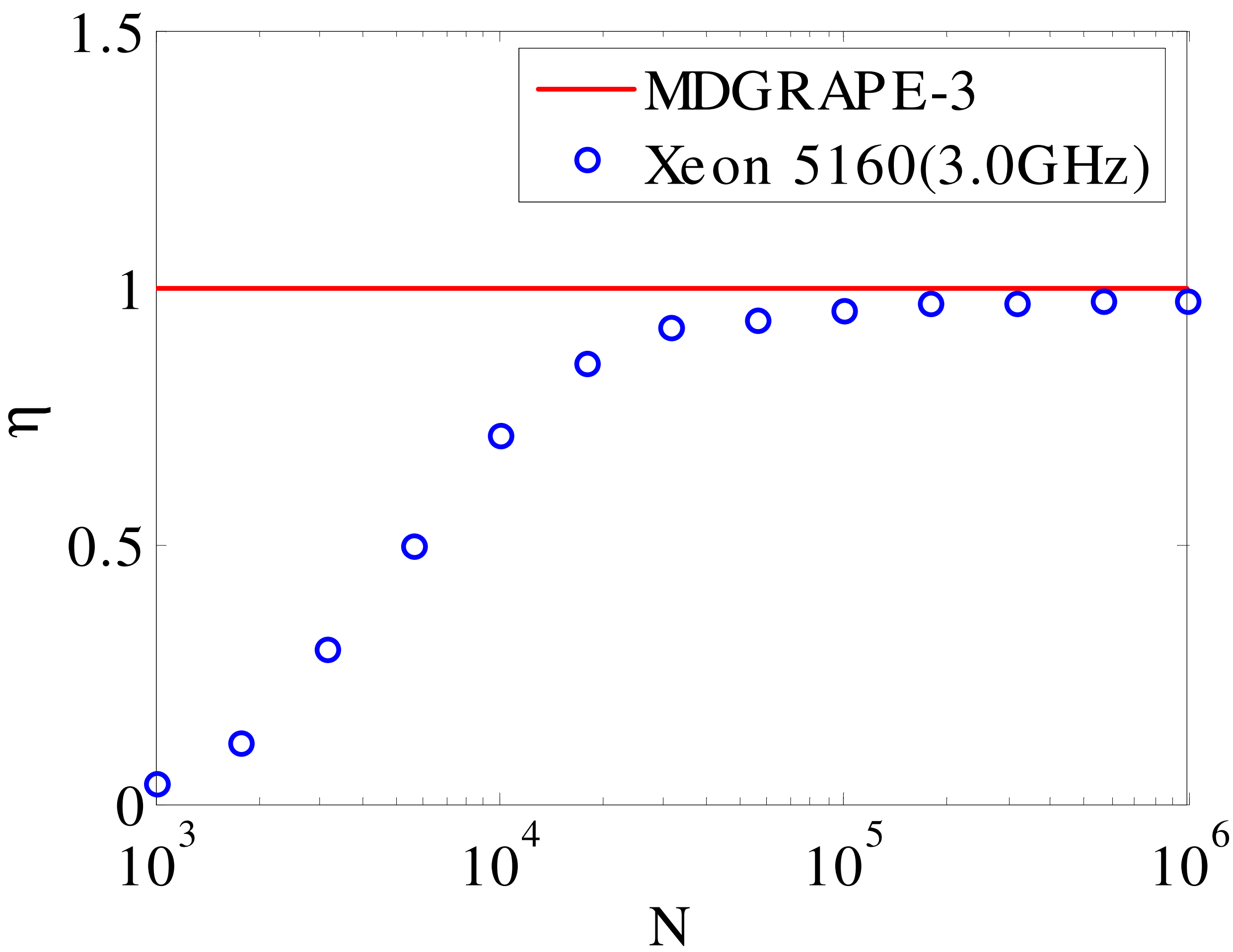
1000 times at N~ 10^6

Efficiency of VM Calculation

$$N_{APPL} = \frac{nmd(xmd + ymd + zmd)}{CPUtime(\mathrm{sec}/\,step)}; \quad N_{GRAPE} = 10^{10}$$

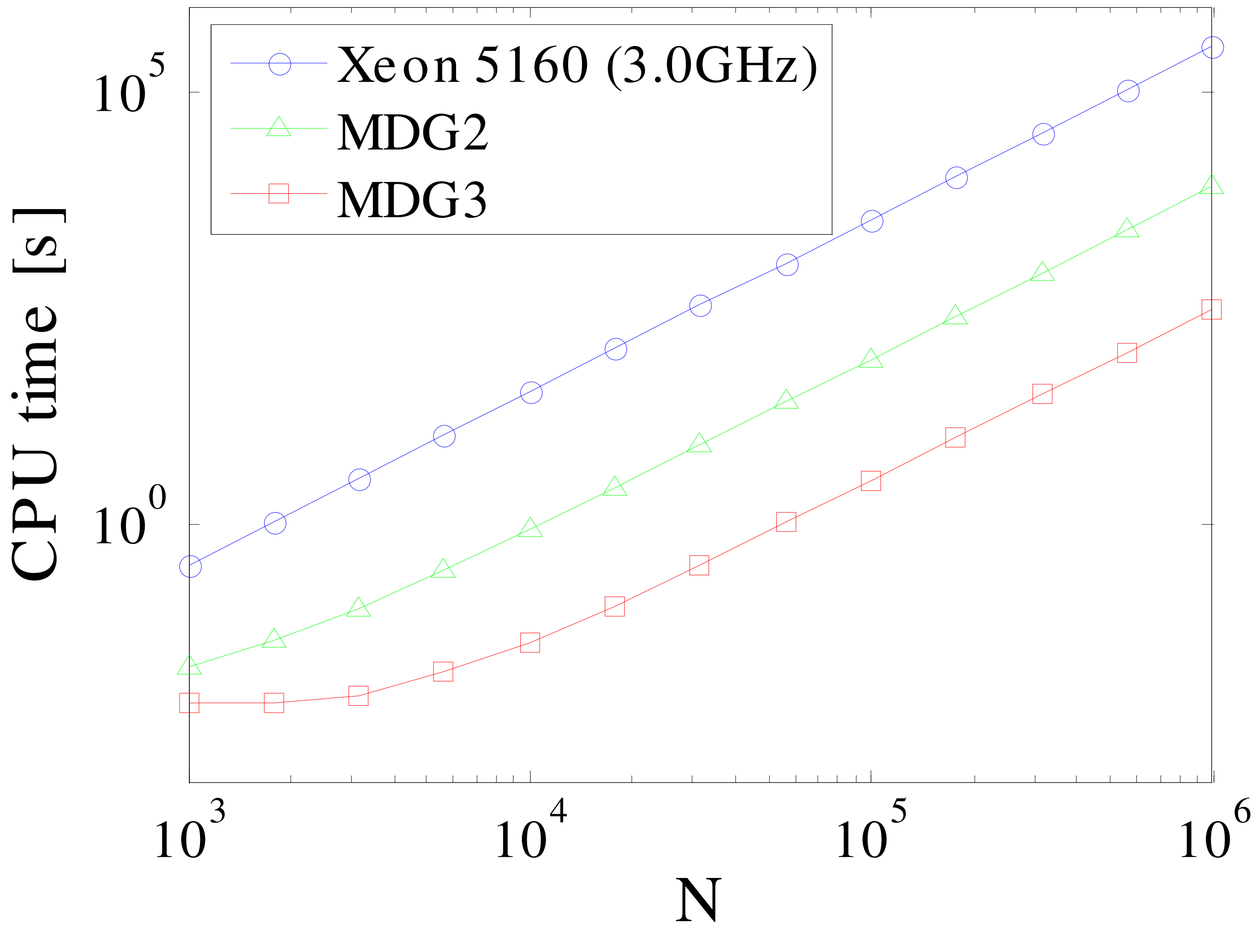
Efficiency:

$$\eta = \frac{N_{APPL}}{N_{GRAPE}}$$

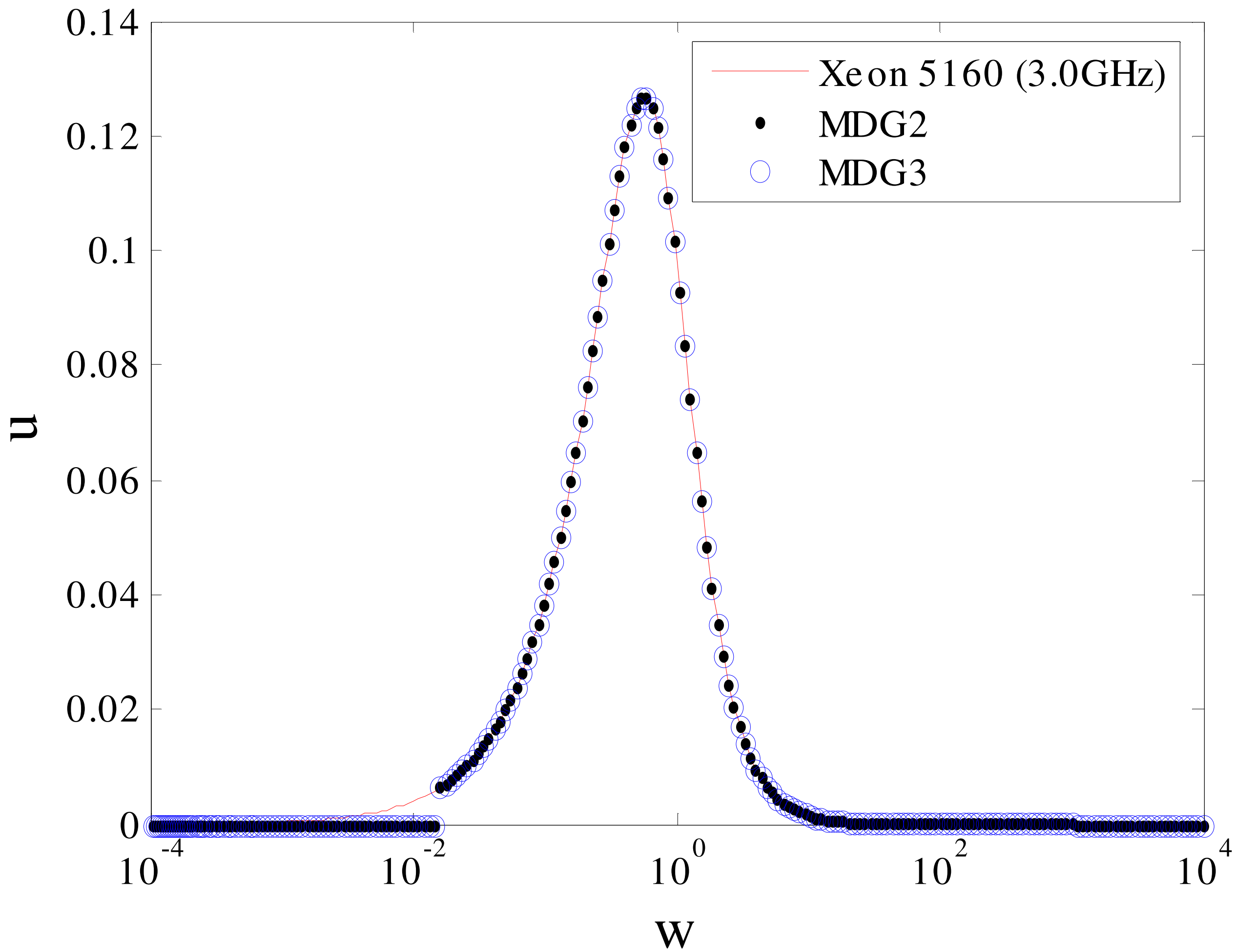


Comparative study between MDGRAPE-2 and MDGRAPE-3

CPU-Time

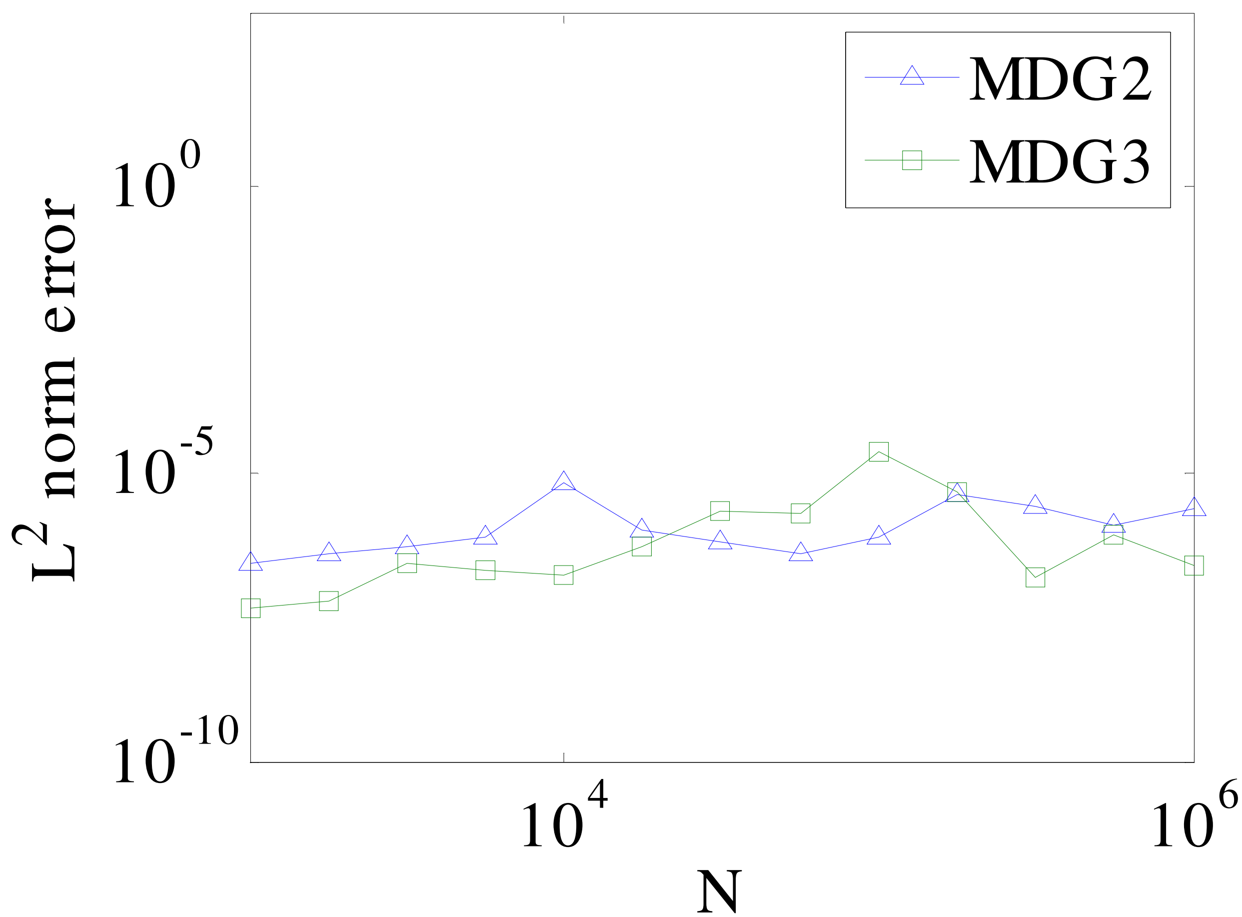


Scaling Error



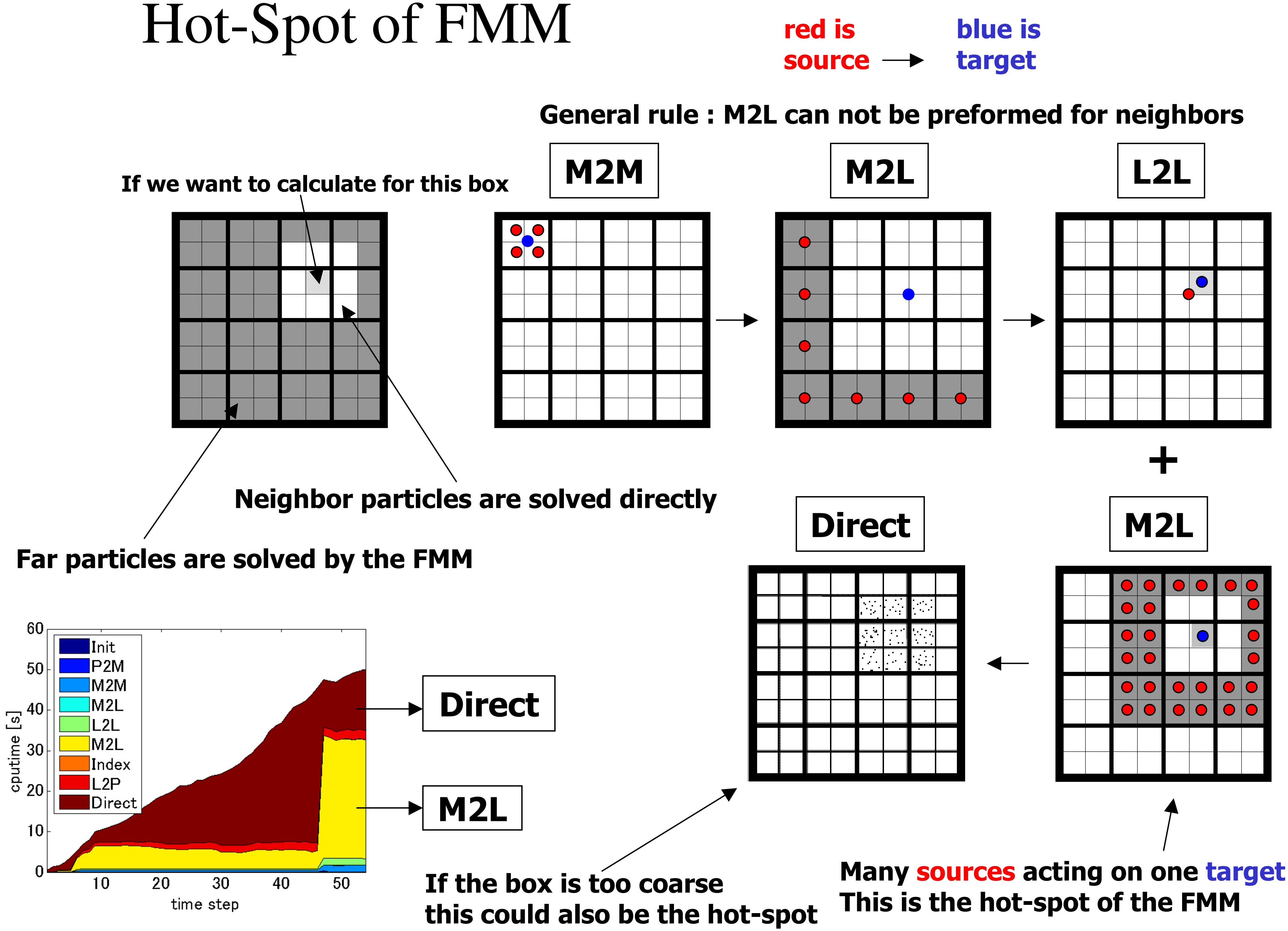
L^2 norm error

$$L^2 = \frac{\sum_j \left\{ (u_{host} - u_{md})^2 + (v_{host} - v_{md})^2 + (w_{host} - w_{md})^2 \right\}}{\sum_j \left\{ u_{host}^2 + v_{host}^2 + w_{host}^2 \right\}} \quad (3.14)$$



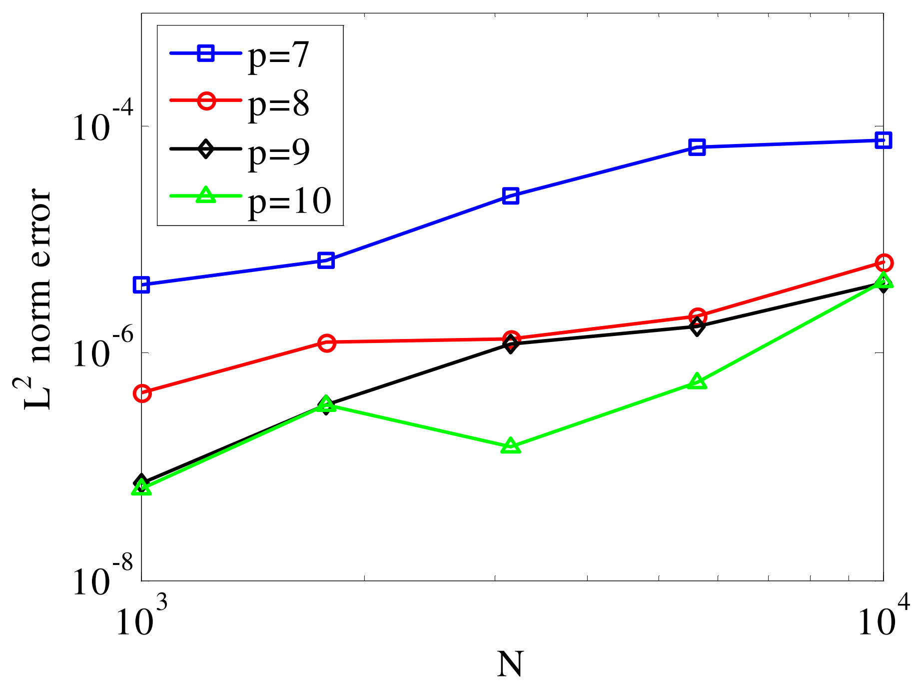
FMM on MDGRAPE-3

Hot-Spot of FMM

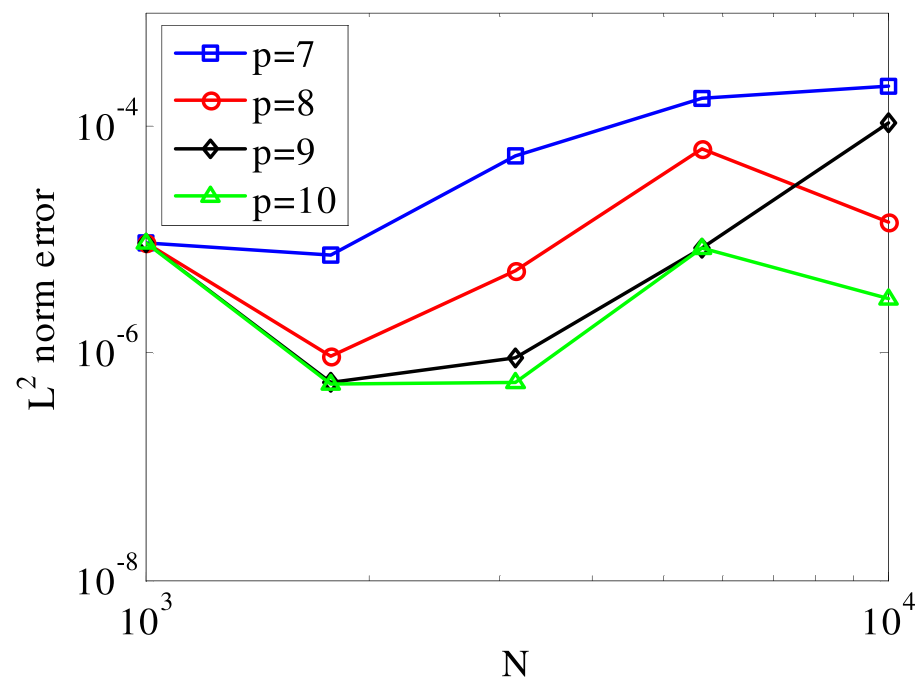


Momentum Effect on FMM Accuracy

$$\mathbf{u}_i \approx \frac{1}{4\pi} \sum_{n=0}^{p} \sum_{m=-n}^{n} \left\{ \sum_{j=1}^{N} \gamma_j M_j \right\} \times \nabla S_i \qquad (2.13)$$



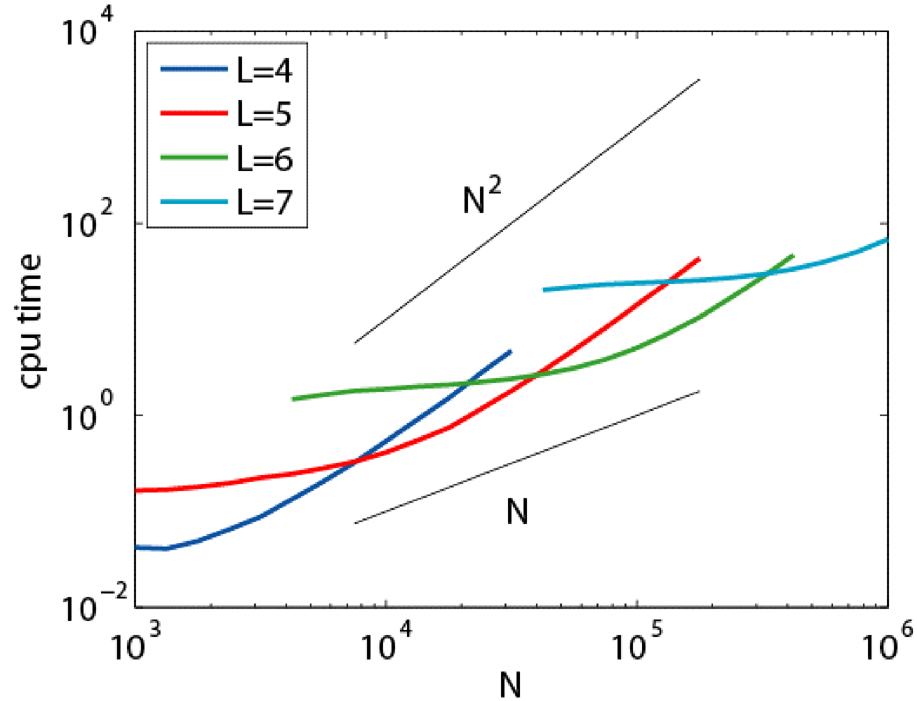
Biot-Savart



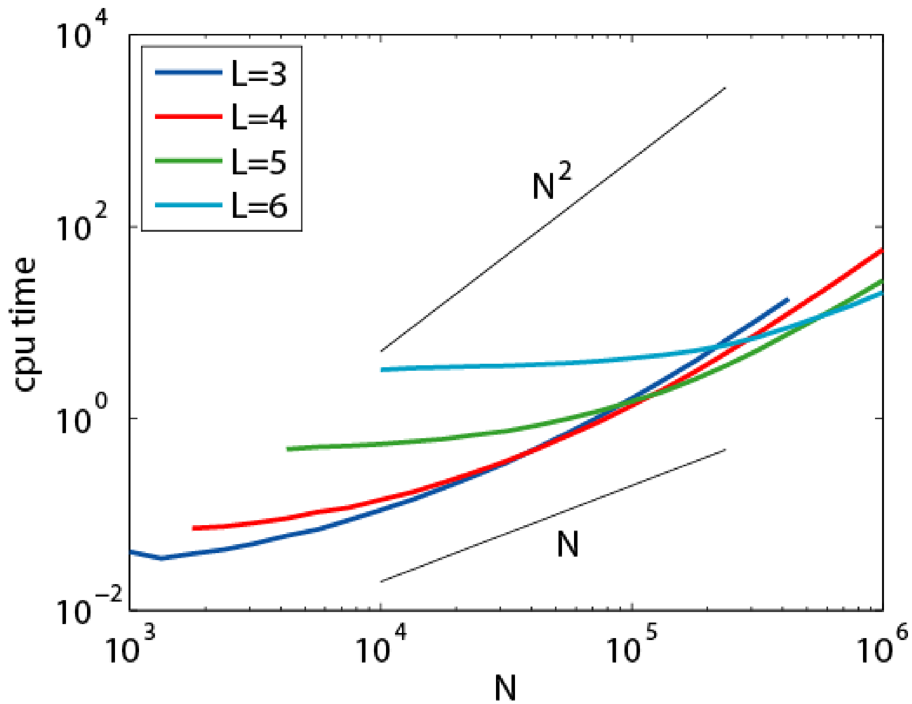
Stretching

5.4.1 Optimization

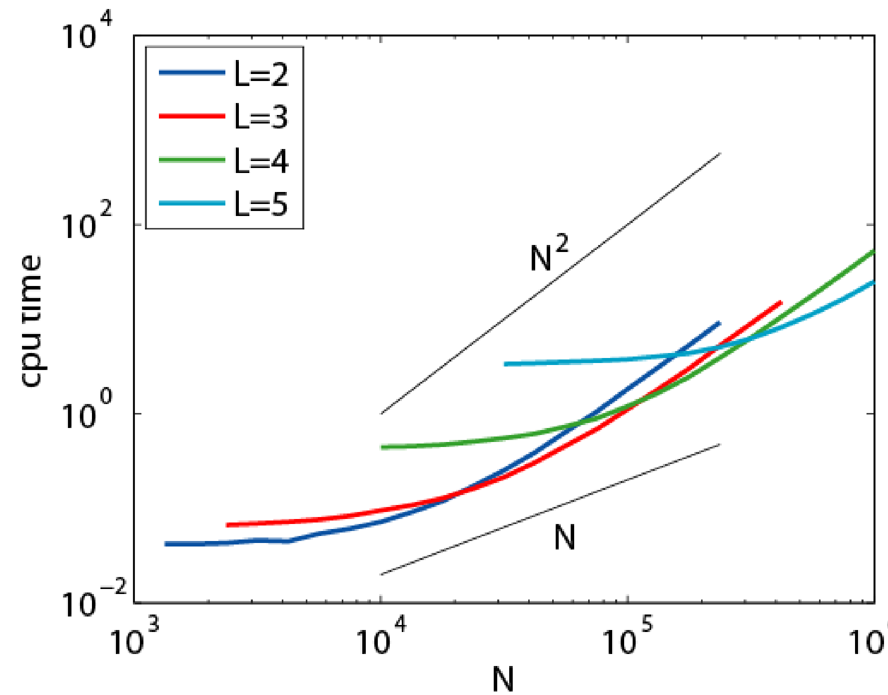
FMM



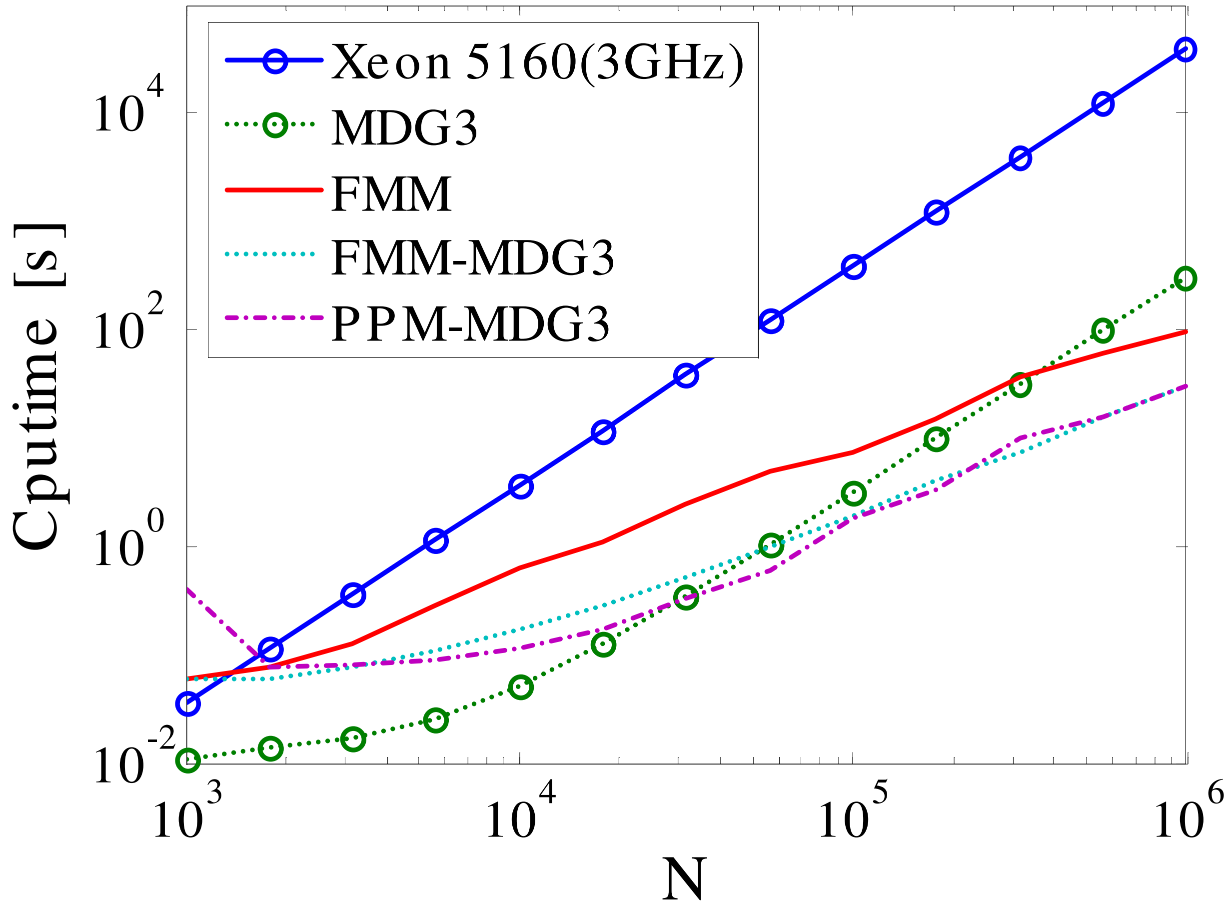
FMM-MDG3

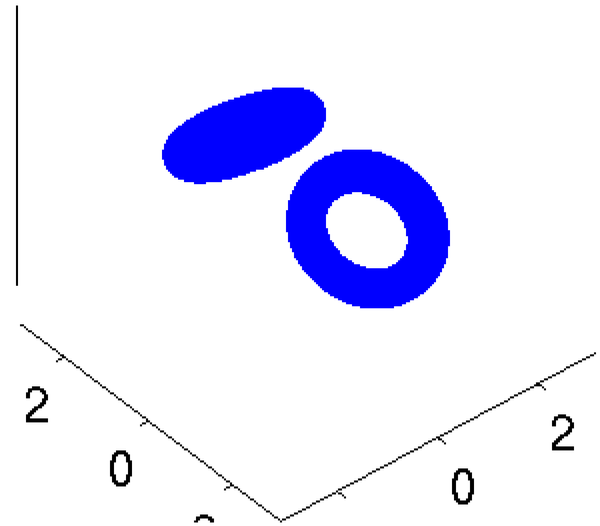


PPM-MDG3



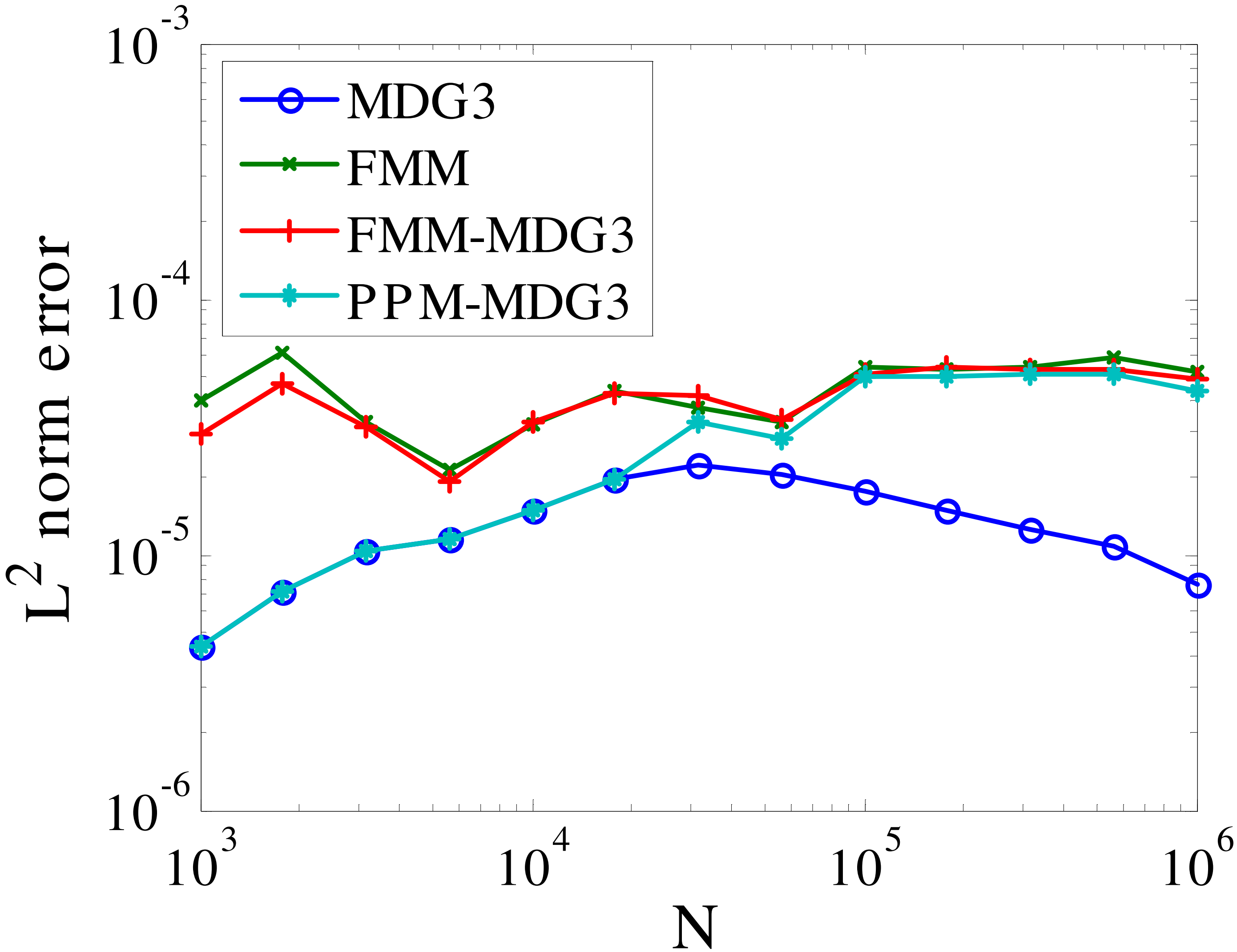
5.4.2 CPU-Time





$N=10^6$

5.4.2 Error



MDG3
FMM
FMM-MDG3
PPM-MDG3
L^2 norm error
N
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{3}
10^{4}
10^{5}
10^{6}

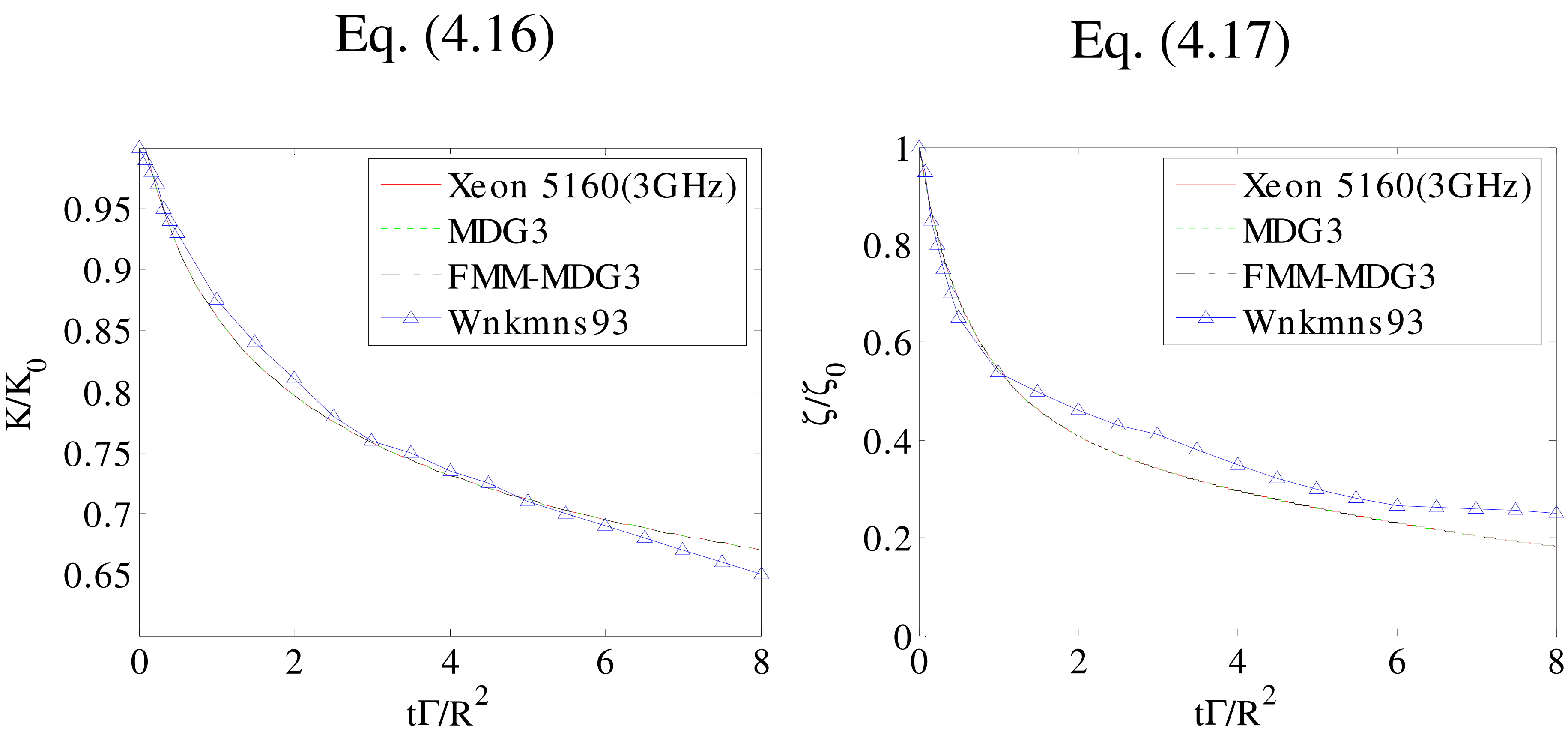
Acceleration Ratio

N=10^6

Biot-Savart		Stretching	
Direct		Direct	
↓ ×462	↓ ×119	↓ ×613	↓ × 52
FMM	MDG3	FMM	MDG3
↓ ×4.1	↓ ×16	↓ ×2.8	↓ ×33
FMM+MDG3		FMM+MDG3	

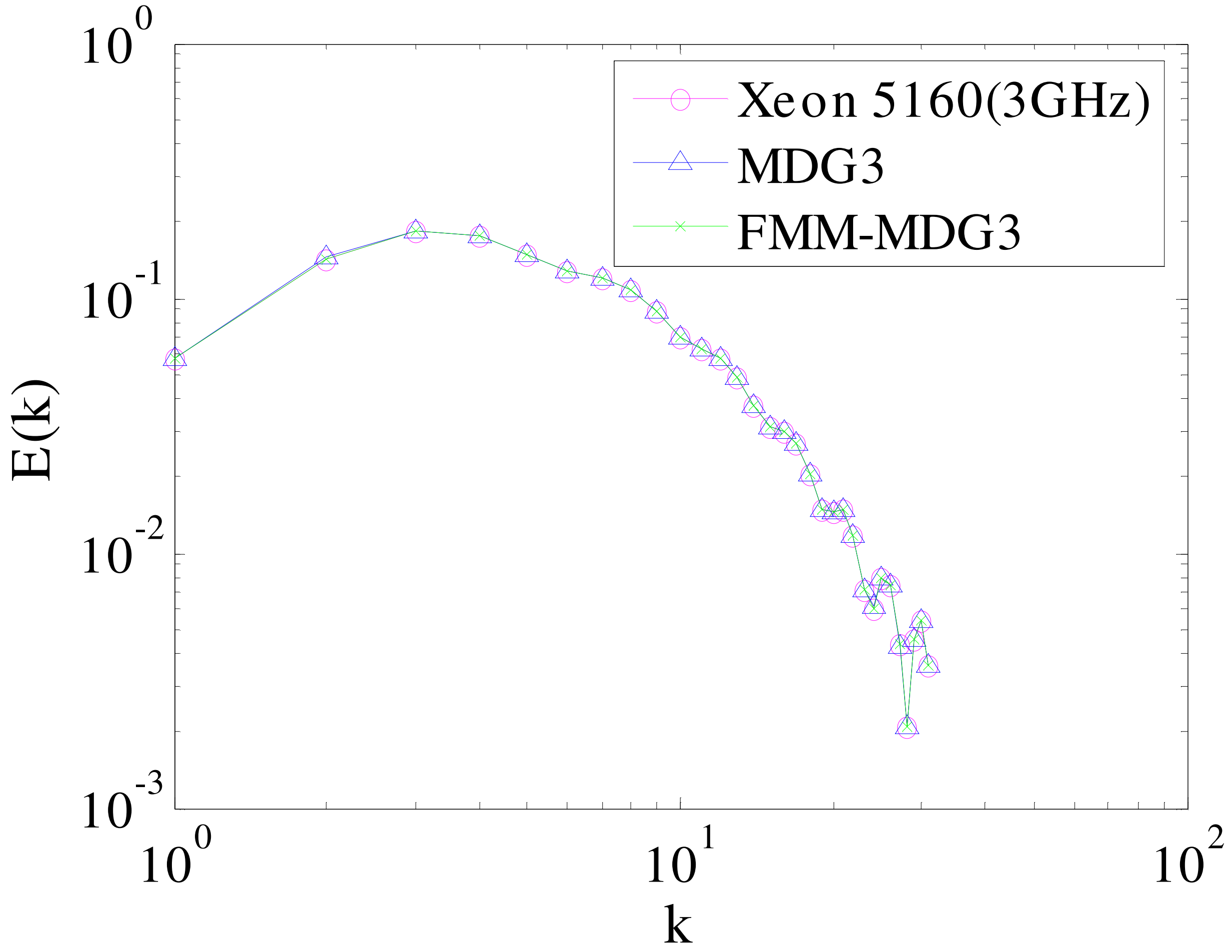
5.5 Vortex Ring Calculation

Energy and Enstrophy



Wnkmns93: G. S. Winckelmans and Leonard, J. Comp. Phys, 109, 247-273(1993)

Energy Spectra

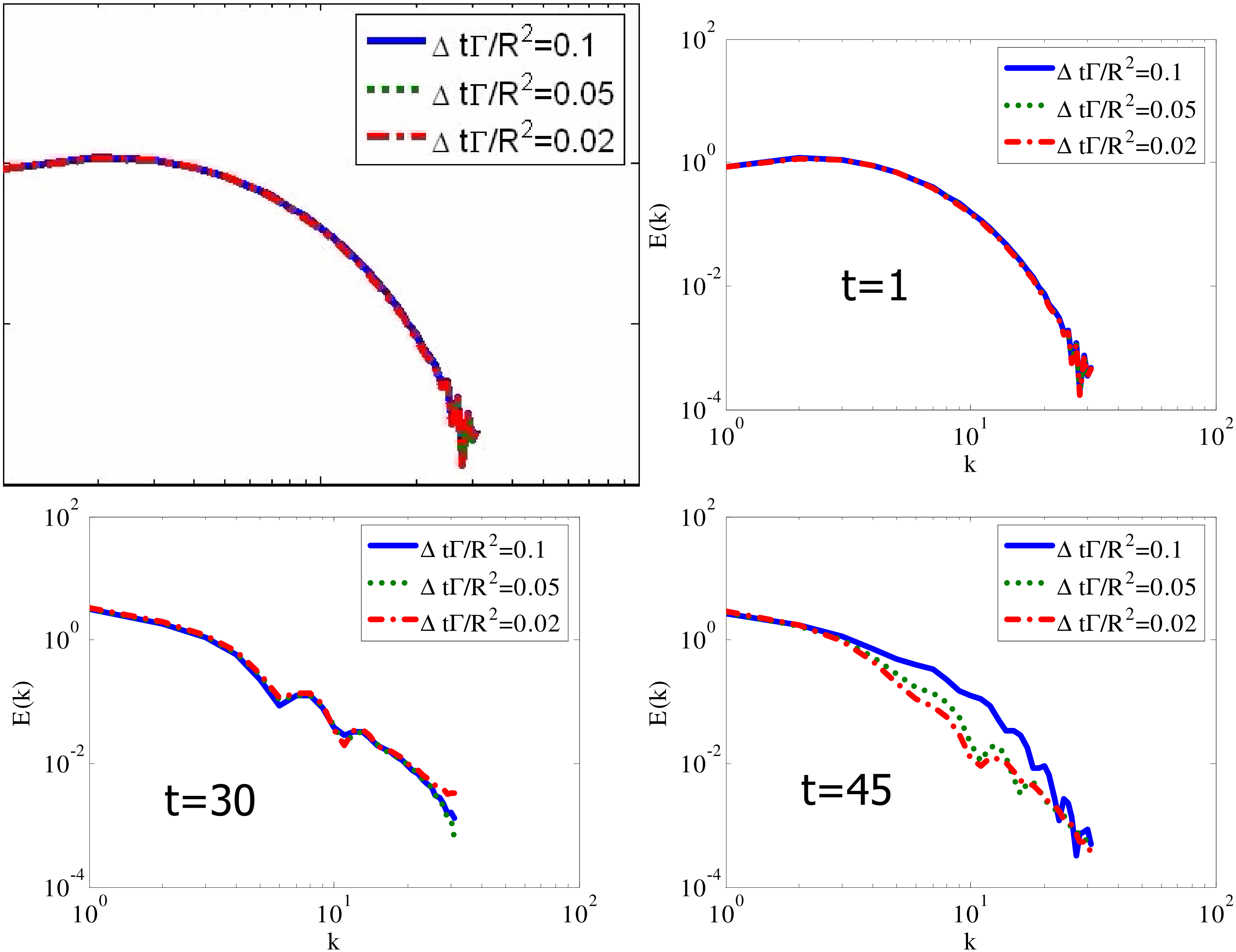


5.5.3 Calculation Conditions

- Vorticity distribution: $$\boldsymbol{\omega} = \frac{\Gamma}{2\pi\sigma^2} \exp\left(\frac{-\mathbf{r}^2}{2\sigma^2}\right) \quad (5.2)$$
- Initial core radius: $$\frac{\sigma_0}{h} = 2 \quad (5.3)$$
- Kinetic Energy: $$K = \frac{1}{2}\sum_{i}^{N} \mathbf{u}_i \cdot \mathbf{u}_i \quad (5.4)$$
- Enstrophy: $$\zeta = \sum_{i}^{N} \boldsymbol{\omega}_i \cdot \boldsymbol{\omega}_i \quad (5.5)$$
- Reynolds number: $$\mathrm{Re}_\Gamma = \frac{\Gamma}{\nu} = 400$$
- Number of particles: $$N = 10^5 \sim 10^7$$

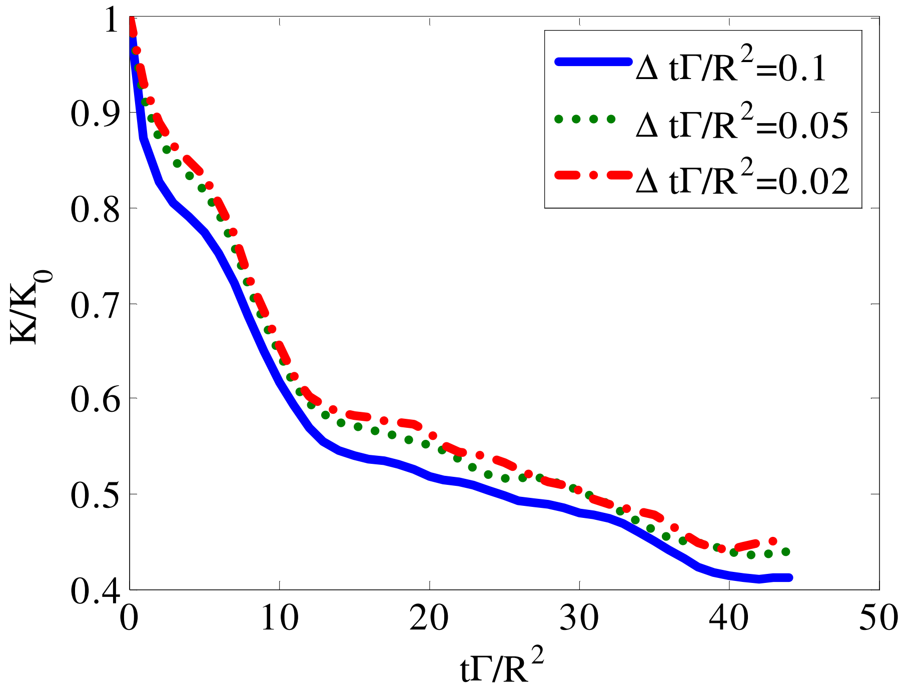
5.5.4 Effect of Temporal Resolution

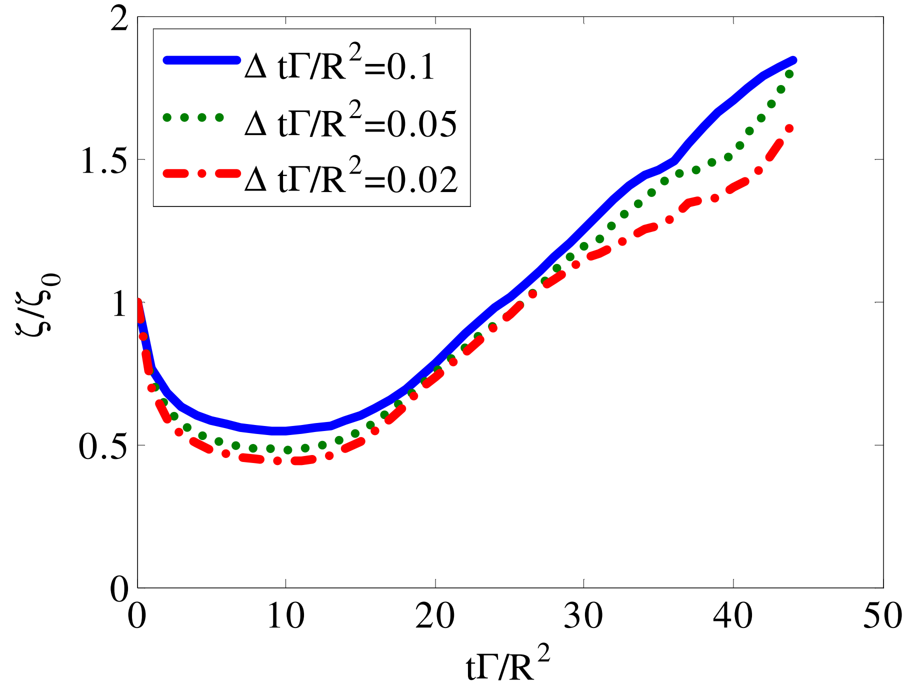
$$\Delta t\Gamma / R^2 = 0.1,\ 0.05,\ 0.02$$



Δ tΓ/R²=0.1
Δ tΓ/R²=0.05
Δ tΓ/R²=0.02
t=1
t=30
t=45
E(k)
k
10^2
10^0
10^-2
10^-4
10^1

Kinetic Energy and Enstrophy (dt)





5.5.5 Effect of Spatial Resolution

Case	A	B	C
Number of Rings	2	2	2
N per Cross Section	190	418	910
Cross Sections	271	1261	5677
Total	102980	1054196	10332140

Position of vortex elements

Ring radius R=1; Cross-section radius r=0.05

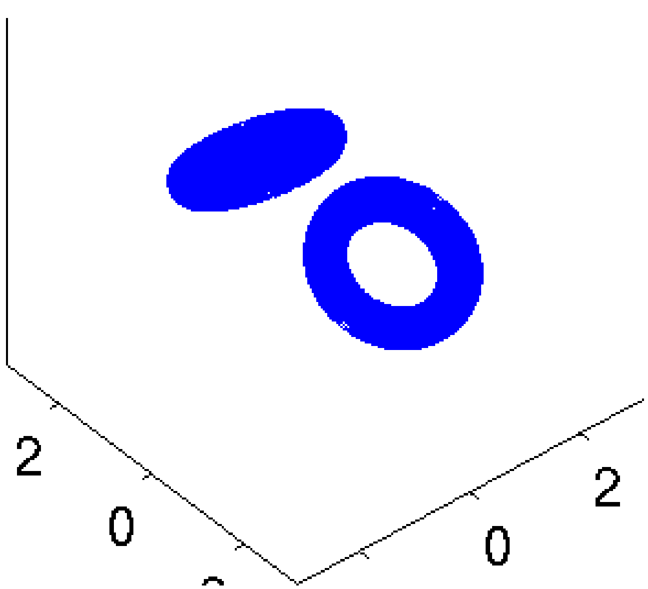
Reynolds number Re=400

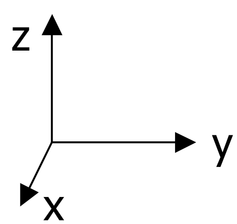
Core radius = 2xspace, Circulation of ring = 1

Initial distance between two rings = 3.0

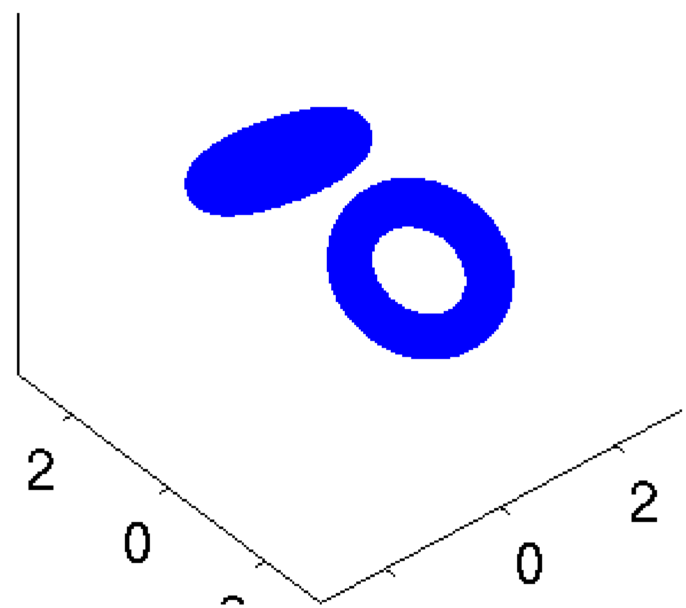
Inclined angle = 30^0

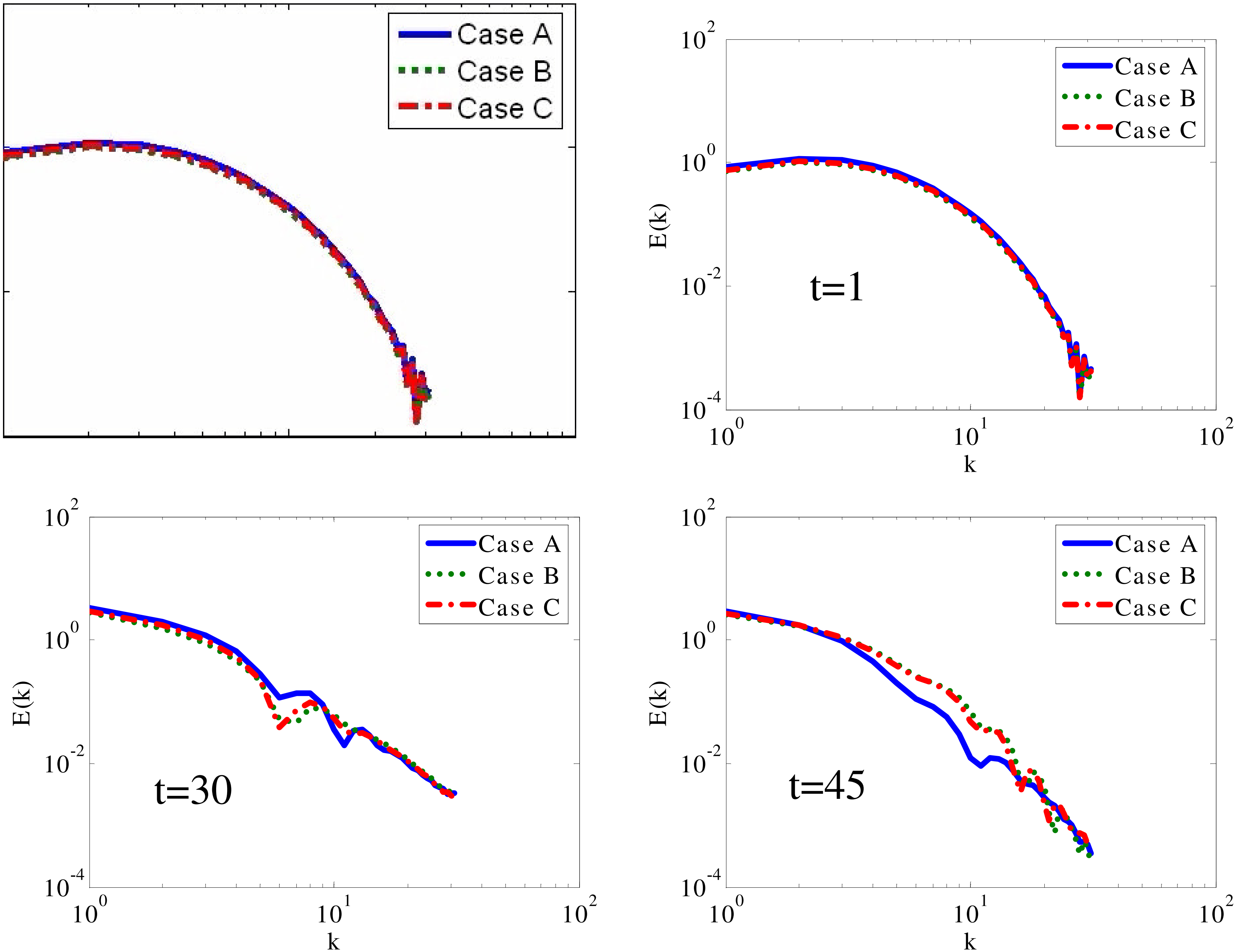
$N=10^5$





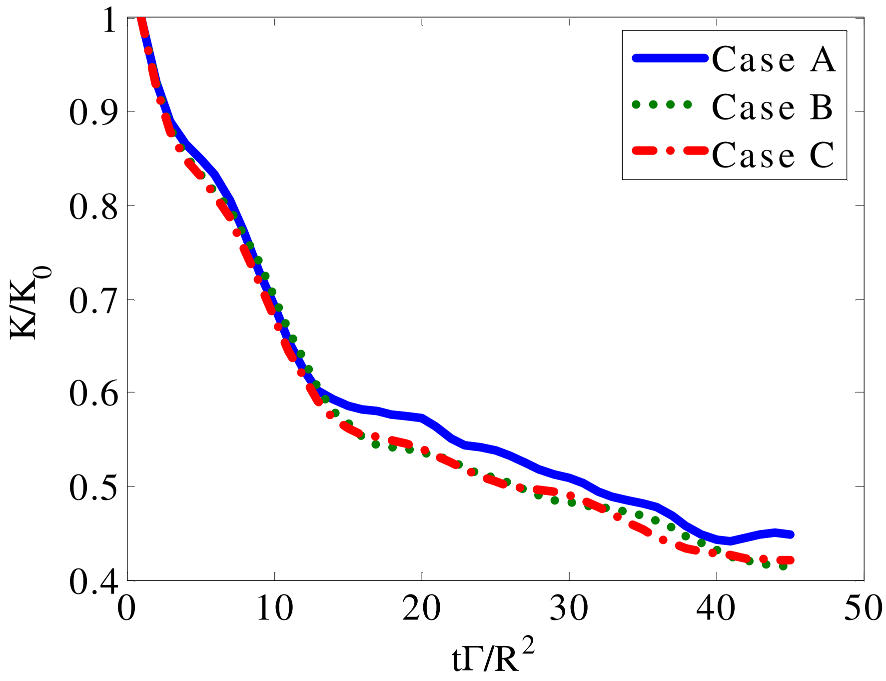
$N=10^6$

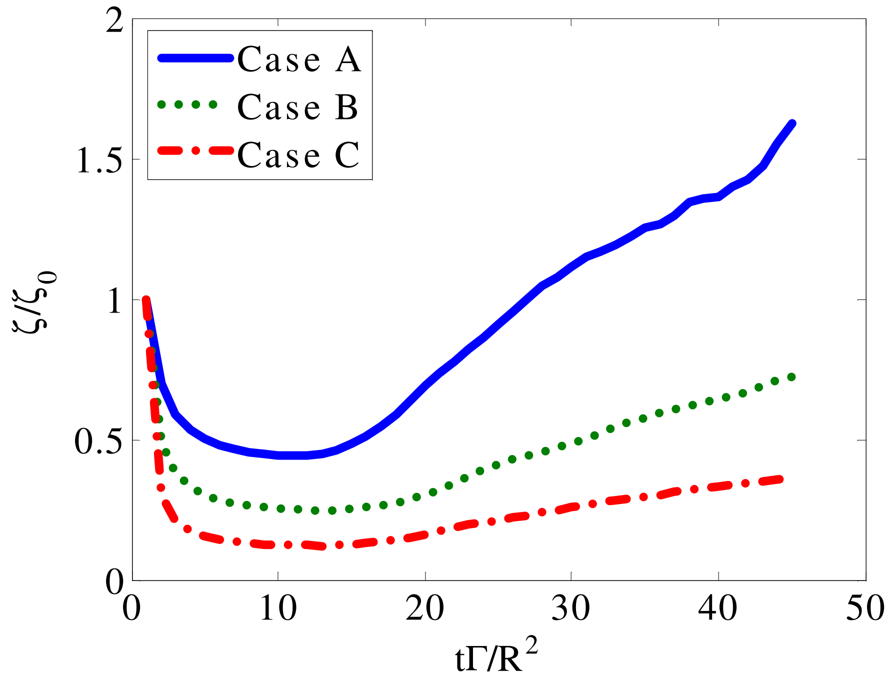




Case A
Case B
Case C
t=1
t=30
t=45
E(k)
k

Kinetic Energy and Enstrophy





Summary

- The vortex method calculation has been accelerated dramatically with the simultaneous use of the FMM and MDGRAPE-3
- Biot-Savart calculation
  - The FMM itself accelerates the calculation 462 times, and the simultaneous use of the MDGRAPE-3 further accelerates it 4.1 times
  - The MDGRAPE-3 can accelerate the calculation 119 times, and the simultaneous use of FMM on MDGRAPE-3 is about 16 times than that of MDGRAPE-3
- Stretching term calculation
  - The calculation cost has been reduced 613 times when used FMM, and another 2.8 times faster by the simultaneous of FMM and MDGRAPE-3
  - The MDGRAPE-3 accelerates 52 times and another 33 times when combined with the FMM
- The errors involved in the use of MDGRAPE-3 are less than the errors of the FMM, and small enough to perform an accurate VM calculation
- The effect of temporal and spatial resolutions are important for accurate calculations

6
Conclusions and Outlook

Conclusion

- A fast vortex method has been developed using special-purpose computers and the simultaneous use of FMM and MDGRAPE-3
- The calculation cost has been reduced significantly by using the proposed acceleration techniques
- The reconnection of the vortex rings was clearly observed, and the discretization error became nearly negligible for the calculation using 10^7 elements
- The overall accuracy are satisfactory for VM calculations

Outlook

- The present results indicate that the calculation of further Reynolds number, the accurate vortex methods requires significantly larger N, which is possible by using the present acceleration method
- There are still some rooms to improve the acceleration rate
  - By reconstructing the subroutines which call MDGRAPE library
  - Present routines require to call MDGRAPE library in 18 times for one cross product term calculation
  - The acceleration can be improved by reducing the CALLing times
- The overall accuracy can be improved by regenerated a sophisticated function table for respective problems
- Other than the present flow, this method can be applied to calculate the homogeneous shear flow, smoothed particle hydrodynamics, dissipative particle dynamics and so on.