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Abstracts of Talks

Cellular Covers of Mixed Abelian Groups

Ulrich Albrecht

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Abstract

A *cellular exact sequence* for an Abelian group A is an exact sequence

$$0 \to K \to G \xrightarrow{\gamma} A \to 0$$

for which the induced map $\gamma_*\colon \mathrm{Hom}(G,G) \to \mathrm{Hom}(G,A)$ is an isomorphism. Every group A admits a cellular exact sequence $0 \to 0 \to A \xrightarrow{\gamma} A \to 0$ with γ an automorphism of A, called the trivial cellular cover. We show that large classes of non-splitting mixed groups have no non-trivial cellular covering sequences answering a question posed by R. Göbel.

Parabolic Lie algebras are Zero Product Determined

Daniel Brice

Tuskegee University

(jointly with Huajun Huang, Auburn University)

Abstract

An algebra, $(A, *)$ is said to be *zero product determined* if for every bilinear map $\varphi : A \times A \to X$ (with X an arbitrary vector space) satisfying $\varphi(x,y) = 0$ whenever $x * y = 0$ there is a linear map $\tilde{\varphi} : A^2 \to X$ such that $\varphi(x,y) = \tilde{\varphi}(x * y)$. Let $\mathfrak{q}$ be a parabolic subalgebra of a reductive Lie algebra $\mathfrak{g}$. Building on the results of D. Wang, et al, and the previous work of B– and Huang, we show that $\mathfrak{q}$ and $\mathrm{Der}\,\mathfrak{q}$ are zero product determined, including the special case where $\mathfrak{q} = \mathfrak{g}$.

Generalized Frobenius-Schur Indicators and Kuperberg 3-manifold Invariants

Liang Chang

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Abstract

Frobenius-Schur indicators were defined originally for finite groups and generalized for Hopf algebras. They are examples of gauge invariants for Hopf algebras, which are useful for the category of representations. Recently, the generalized indicators turned out to coincide with Kuperberg 3-manifold invariants for Lens spaces, which provides topology interpretation for Hopf algebra invariants. In this talk, I will explain these algebraic and topological invariants and recent work on their relation.

Groups with the Weak Minimal Condition on Non-Permutable Subgroups

Laxmi Kant Chatuat

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Abstract

Let H be a subgroup of a group G. Then H said to be *permutable* if it permutes with every subgroup of G, that is, $HK = KH$ for every subgroup K of G. Let $\mathcal{P}$ be a subgroup theoretical property or class of groups, then $\bar{\mathcal{P}}$ is the class of all groups that either are not-$\mathcal{P}$ groups or are trivial. A group G is said to satisfy the *weak minimal condition on $\mathcal{P}$-subgroups* (denoted by min-∞-$\mathcal{P}$) if for every descending chain $H_1 > H_2 > H_3 > \cdots$ of $\mathcal{P}$ subgroups of G, the index $|H_i : H_{i+1}|$ is infinite for only finitely many i. Thus, for example, on letting $\mathcal{P}$ denotes the class of permutable subgroups, we may speak of groups satisfy min-∞-$\bar{\mathcal{P}}$, the weak minimal condition on non-permutable subgroups. Groups with this property are the subject of our interest.

The main results are as follows; If G is a locally finite group satisfying the weak minimal condition on non-permutable subgroups then either G is Chernikov or every subgroup of G is permutable. It is also proved that for a generalized radical group G satisfying the weak minimal condition on non-permutable subgroups either G has *finite rank* or every subgroup of G is permutable.

Quantum Computation and Symmetric Spaces

Jennifer Daniel

Lamar University

Abstract

Quantum computation involves evolution of state vectors under unitary transformations. A *quantum circuit* on n qubits is a unitary mapping on H^{2^n}, which can be represented as a concatenation of a finite set of quantum gates. The ability to decompose a quantum circuit into the product of simple quantum gates is essential to the design of a quantum computer. In this work, we utilize the root space decomposition of a local symmetric space of type-**AIII** to decompose a quantum circuit into the tensor product of unary and controlled NOT gates.

Localizations of Tensor Products

Manfred Dugas

Baylor University

Abstract

Given suitable R-modules A and B, it is well known how to define their tensor product $A \otimes_R B$. We call the module T a qutensor product of A and B if there is some middle linear map $\tau : A \times B \to T$ such that for all middle linear maps $\sigma : A \times B \to T$ there is a unique $\varphi \in End(T)$ such that $\sigma = \varphi \circ \tau$. It turns out that qutensor products are localizations of the standard tensor product. We will investigate their properties and construct examples.

This is joint work with K. Aceves and B. Wagner.

The Number of Irreducible Representations of an Augmented Associative Algebra and the Projective Cover of the Trivial Module

Jörg Feldvoss

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Abstract

In this talk I will discuss a relationship between the projective cover of the trivial irreducible module of a finite-dimensional augmented associative algebra and the number of isomorphism classes of its irreducible representations. This is motivated by recent joint work with Salvatore Siciliano and Thomas Weigel on restricted Lie algebras.

Derivations of the Lie Algebra of Dominant Upper Triangular Ladder Matrices

Prakash Ghimire

Auburn University

Abstract

Let M_n be the general Lie algebra consisting of all $n \times n$ matrices over a characteristic zero field $\mathbb{F}$, and $M_{\mathcal{L}}$ be the subalgebra of M_n consisting of all matrices corresponding to a dominant upper triangular ladder $\mathcal{L}$. Then any derivations of $M_{\mathcal{L}}$ can be expressed as the sum of $\mathrm{ad}(X)$, where X is a block upper triangular matrix in M_n, and a linear transformation mapping $[M_{\mathcal{L}}, M_{\mathcal{L}}]$ to zero, and $M_{\mathcal{L}}$ to $Z(M_{\mathcal{L}}) \cap M_{\mathcal{L}}$, where $Z(M_{\mathcal{L}})$ is the center of $M_{\mathcal{L}}$ in M_n.

On the Question of Which Torsion-free Abelian Groups of Rank Two Possess FI-Extending Hulls

Pat Goeters and Gary Birkenmeier

Auburn University and University of Louisiana at Lafayette

Abstract

An abelian group H is called FI-extending, if every fully invariant submodule of H is contained as an essential submodule of a summand of H. A module G is said to have an *FI-extending* H if

(i) H is FI-extending, and
(ii) If K is FI-extending with $G < K < H$, then $K = H$.

While every torsion group has an FI-extending hulls, such is not the case for torsion-free groups. We characterize the rank 2 torsion-free groups having FI-extending hulls.

The question of whether a rank 2 groups possesses an FI-extending hull can be answered by breaking the problem into two collections; the homogeneous groups and the non homogeneous groups. In the instance of homogeneous groups, the group has an FI-extending hull exactly when the endomorphism ring is great than or equal to 2. This in turn is equivalent to the group being irreducible (FI-extending). The main point here is whether or not there exist homogeneous groups whose endomorphism ring is trivial. We provide an example of a homogeneous group which endomorphism ring is $\mathbb{Z}$, and to our understanding, such an example does not appear in the literature.

In the nonhomogenous case, the group G has an FI-extending exactly when G is quasi-isomorphic to $X_1 \oplus X_2$, with X_1 and X_2 incomparable in the sense of types. Notice that the groups quasi-isomorphic to, but not isomorphic to $X_1 \oplus X_2$, with X_1 and X_2 incomparable, are the only examples of non FI-extending groups with FI-extending hulls. To complete the non homogeneous case, we must prove the result that $G = X \oplus Y$ with $X \leq Y$ and X, Y incomparable, does not have an FI-extending hull.

$\aleph_k$-free Cogenerators

Daniel Herden

Baylor University

Abstract

An abelian group G is called $\aleph_k$-free if every subgroup $U \subseteq G$ of cardinality $< \aleph_k$ is free. While the existence of non-trivial $\aleph_1$-free groups in ZFC is a well known result of Baer-Speker, the existence of non-trivial $\aleph_k$-free groups $(k > 1)$ in ZFC could be established only recently and the properties of $\aleph_k$ -free groups are still vastly unknown. In this talk we would like to discuss the relation of $\aleph_k$-free groups to cotorsion groups, i.e., groups C with $\operatorname{Ext}(\mathbb{Q}, C) = 0$. Due to a result by Hunter a group G is cotorsion if and only if $\operatorname{Ext}(G, C) = 0$ for all $\aleph_1$-free groups G. We would like to show that this result is still true while replacing $\aleph_1$-free by the much stronger notion $\aleph_k$-free $(k > 1)$.

Eisenstein-Dumas Criterion and the Action of 2×2 Non-Singular Triangular Matrices on Polynomials in One Variable

Martin Juras

Qatar University

Abstract

In the past 150 years, original criteria of Schönemann and Eisenstein were generalized in many different ways. One far reaching generalization comes in the form of a geometric criterion of Dumas:

Newton polygon of a product of polynomials is composed from the sides of polygons of its factors which are ordered with respect to increasing slopes.

In case when the Newton polygon of a polynomial consists of only one line with no interior lattice points, Dumas' criterion implies irreducibility. This special case is called Eisenstein-Dumas criterion.

Let K be a valued field with valuation v and $A(x) \in K[x]$ be a polynomial of degree n. We find necessary and sufficient conditions for the existence of the elements $s, t, u \in K$, $s \neq 0 \neq u$, such that at least one of the polynomials $u^n A(\frac{sx+t}{u})$, $(tx+u)^n A(\frac{sx}{tx+u})$, $(ux)^n A(\frac{tx+s}{ux})$ or $(ux+t)^n A(\frac{s}{ux+t})$ is an Eisenstein-Dumas polynomial at v, provided that the characteristic of the residue field of v does not divide n. We show that if the orbit $A(x)GL(2, K)$ contains an Eisenstein-Dumas polynomial at v, then an Eisenstein-Dumas polynomial can be found in a certain one-parameter subset of $A(x)GL(2, K)$.

Groups with all Subgroups Permutable or Soluble

Zekeriya Y. Karatas

Tuskegee University

(Joint with Martyn R. Dixon)

Abstract

In this talk, I will give the structure of locally graded groups with all subgroups permutable or soluble of bounded derived length. Such a group is soluble of given derived length for the subgroups, or its second derived subgroup is finite and perfect with all subgroups soluble of bounded derived length. This theorem will imply that locally graded groups with all subgroups permutable or nilpotent of bounded class are soluble of bounded derived length depending on the nilpotency class. Before giving the results, I will give the definitions, well-known results and some history about the concept.

Standard Vector Bundles

Youngsoo Kim

Tuskegee University

Abstract

The construction of the category of standard vector bundles over a scheme X will be described. It is the category that is equivalent to the usual category of vector bundles (i.e., locally free $\mathcal{O}_X$-modules of finite rank) on the scheme, but it has additional properties such as strict associativity of tensor product, strict commutativity with line bundles, and strict functoriality on base change among others.

Matrix Algebras over Strongly Non-Singular Rings

Brad McQuaig

Auburn University

Abstract

We consider some existing results regarding rings for which the classes of torsion-free and non-singular right modules coincide. Here, a right R-module M is *non-singular* if xI is nonzero for every nonzero $x \in M$ and every essential right ideal I of R, and a right R-module M is *torsion-free* if $\mathrm{Tor}_1^R(M, R/Rr) = 0$ for every $r \in R$.

In particular, we consider a ring R for which the classes of torsion-free and non-singular right S-modules coincide for every ring S Morita-equivalent to R. We then look to characterize rings whose $n \times n$ matrix rings are Baer-rings.

A ring is *Baer* if every right (or left) annihilator is generated by an idempotent.

Rings which are semi-hereditary, strongly non-singular, and right and left Utumi will play an important role, and we discuss relevant results as well as some examples of rings which fail one of these conditions.

Covering Ideals of Morphisms

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Abstract

A significant result of cotorsion theory proven by Eklof & Trlifaj [1] is that if (F, C) is cogenerated by a set, then it is complete. Recently the cotorsion pairs of ideals (I, J), where I and J are subfunctors of Hom_R, have been of interest [2]. In this talk we will look at a few results motivated by Eklof & Trlifaj argument for an ideal I when it is generated by a set. Moreover, we will show how identifying an ideal I with a certain class of objects in A_2 (category of all representations of 2-quiver by modules) can help us to obtain sufficient conditions for I to be a covering ideal.

References

[1] P.C. Eklof, J. Trlifaj, *How to make Ext vanish*, Bull. London Math. Soc. **33** (2001), 31–41.

[2] X.H. Fu, P.A. Guil Asensio, I. Herzog, and B. Torrecillas. *Ideal approximation theory.* Adv. Math. **244** (2013) 750–790.

Lifting Modules for a Finite Group of Lie Type to its Ambient Algebraic Group

Cornelius Pillen

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Abstract

Let p be a prime and q a prime power. The algebraic closure of the field $\mathbf{F}_q$ with q elements is denoted by k. A Zariski closed subgroup G of $\mathrm{GL}_n(k)$, the general linear group with entries in k, is an algebraic group. If we replace the field k by one of the finite fields $\mathbf{F}_q$ we obtain a finite group of Lie type, denoted by $G(q)$, sitting inside the infinite group G.

We are interested in the following question: Can a module of the finite group be lifted to a module for the algebraic group G? For example, a well-known result of Robert Steinberg says that all the simple modules can be lifted. But in general the answer to the aforementioned question is no. This talk is a survey of known results together with some explicit SL_2 examples.

On Yangians and Finite W-algebras

E. Poletaeva

University of Texas–Pan American

Abstract

Finite W-algebras are certain associative algebras attached to a pair $(\mathfrak{g}; e)$, where $\mathfrak{g}$ is a complex semisimple Lie algebra or a classical Lie superalgebra and $e \in \mathfrak{g}$ is an even nilpotent element. J. Brown, J. Brundan, and S. Goodwin proved that the finite W-algebra for the general linear Lie superalgebra $\mathfrak{gl}(m|n)$ for regular nilpotent e is isomorphic to a truncation of a shifted Yangian of $\mathfrak{gl}(1|1)$.

The Yangian $Y(Q(n))$ of the queer Lie superalgebra $Q(n)$ was defined by M. Nazarov. It is a Hopf algebra. We show that the finite W-algebra for $Q(n)$ for regular nilpotent e is isomorphic to a quotient of $Y(Q(1))$, and describe this isomorphism using the comultiplication on $Y(Q(1))$, evaluation homomorphism, and Harish-Chandra homomorphism. We observe that certain even generators of $Y(Q(1))$ commute, and use this fact to construct generators in the finite W-algebra for $Q(n)$.

It is a joint work with V. Serganova.

Maximal Prime Products Arising From Bertrand Primes

Kenneth J. Prevot

Metropolitan State University of Denver

Abstract

For integers $n > 1$, the Bertrand Conjecture states that there exists a prime p above the middle dimension, namely that $n < p < 2n$. This was proven by Chebyshev in 1850. For a fixed integer n, primes p of this sort will be referred to as Bertrand primes. Starting with a Bertrand prime p, one may form the product pq where q is the largest odd prime factor in the prime factorization of the difference $(2n - p)$. Certain results and observations regarding the product pq are presented when pq is maximal across the set of Bertrand primes p for a fixed n.

Leavitt Path Algebras Satisfying a Polynomial Identity

Kulumani M. Rangaswamy

University of Colorado, Colorado Springs

Abstract

Leavitt path algebras over arbitrary graphs satisfying a polynomial identity are completely characterized both graphically and algebraically.

Extension of Wang-Gong Monotonicity Result on Positive Definite Matrices to Semisimple Lie Groups

Zachary Sarver and Tin-Yau Tam

Auburn University

Abstract

When x and y are nonnegative, we say that x is *log majorized* by y, denoted by $x \prec_{\log} y$, if

$$\prod_{i=1}^{k} x_{[i]} \le \prod_{i=1}^{k} y_{[i]}, \quad k = 1, 2, \ldots, n-1, \quad \text{and} \quad \prod_{i=1}^{n} x_{[i]} = \prod_{i=1}^{n} y_{[i]}.$$

Here $x^{\downarrow} = (x_{[1]}, x_{[2]}, \ldots, x_{[n]})$ denotes a rearrangement of the components of x such that $x_{[1]} \ge x_{[2]} \ge \cdots \ge x_{[n]}$. Let A, B be $n \times n$ positive semi-definite matrices. Wang-Gong's inequality asserts that

$$\lambda^{1/r}(A^r B^r) \prec_{\log} \lambda^{1/s}(A^s B^s)$$

if $0 < r < s$, in which $\lambda(A)$ denotes the vector of eigenvalues of A.

This is a generalization of some results of Marcus, Lieb and Thirring, le Couteur, and Bushell and Trustrum. We extend Wang-Gong's inequality in the context of semisimple Lie groups.

Triviality Theorems for Yetter-Drinfel'd Hopf Algebras

Yorck Sommerhäuser

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Abstract

Usually, a Yetter-Drinfel'd Hopf algebra is not a Hopf algebra. Yetter-Drinfel'd Hopf algebras that are ordinary Hopf algebras are called trivial; by a result of P. Schauenburg, this happens if and only if the quasisymmetry in the category of Yetter-Drinfel'd modules accidentally coincides with the ordinary flip of tensor factors on the second tensor power of the Yetter-Drinfel'd Hopf algebra.

In certain situations, every Yetter-Drinfel'd Hopf algebra is trivial. In the talk, we consider a finite-dimensional semisimple Yetter-Drinfel'd Hopf algebra A over the group ring $K[G]$ of a finite abelian group G, where K is an algebraically closed field of characteristic zero, and first discuss the following triviality theorem:

If A is commutative and its dimension is relatively prime to the order of G, then A is trivial.

Even if the Yetter-Drinfel'd Hopf algebra is not completely trivial, it sometimes must contain a trivial part, as stated in the following partial triviality theorem:

If A is cocommutative and its dimension is greater than 1, then A contains a trivial Yetter-Drinfel'd Hopf subalgebra of dimension greater than 1.

We also discuss the essential methods needed for the proof of these two results.

Inverse Semigroups Critical with Respect to a Given Collection of Semigroups

Ralph P. Tucci

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Abstract

An *existence variety* or *e-variety* of semigroups is a class of semigoups which is closed under direct products, homomorphic images, and regular subsemigroups. An *inverse semigroup* is a regular semigroup in which the von Neumann regular inverses are unique. In this talk we present some preliminary results describing those inverse semigroups S whose proper homomorphic images are all in some given e-variety V for various V.

Extensions of Finite Group Schemes in Characteristic p

Robert G. Underwood

Auburn University at Montgomery

Abstract

Let R be a complete discrete valuation ring with field of fractions K and $\mathrm{char}(R) = 0$. Assume that R contains a primitive pth root of unity ζ_p. Let C_p denote the cyclic group of order p, and let C_p^2 denote the elementary abelian group of order p^2. Let H_{i_1} and H_{i_2} denote R-Hopf orders in KC_p, with corresponding group schemes $\mathbf{D}_{i_1}$, $\mathbf{D}_{i_2}$. The group of equivalence classes of 1-extensions of $\mathbf{D}_{i_2}$ by $\mathbf{D}_{i_1}$, denoted as $\mathrm{Ext}^1(\mathbf{D}_{i_1}, \mathbf{D}_{i_2})$, has been classified by C. Greither. In this talk, we give the analogous result in characteristic p: We classify $\mathrm{Ext}^1(\mathbf{D}^*_{i_2}, \mathbf{D}^*_{i_1})$, where $\mathbf{D}^*_{i_1}$, $\mathbf{D}^*_{i_2}$ are the Cartier duals of the group schemes $\mathbf{D}_{i_1}$, $\mathbf{D}_{i_2}$. This leads to a complete classification of R-Hopf orders in KC_p^2 in characteristic p.

This is joint work with G. Griffith Elder, University of Nebraska–Omaha.

Finite Groups with Metacyclic QTI-Subgroups

Gary Walls

Southeastern Louisiana University

Abstract

(Joint work with Zhencai Shen, Yingyi Chen, and Shirong Li)

Let G be a finite group. A subgroup H of G is called a QTI-subgroup if $C_G(x) \subseteq N_G(H)$ for all $1 \neq x \in H$. This is a generalization of both H being a normal subgroup and H being a TI-subgroup. In this paper we study groups in which every metacyclic subgroup is a QTI-subgroup. We call such groups MCTI-groups.

We are able to classify all the finite nilpotent MCTI-groups. They are either Dedekind groups or are p-groups for some prime p. We completely classify the MCT-p-groups. We show that all MCTI-groups are solvable and that every finite, nonnilpotent MCTI-group must be a Frobenius group having abelian kernel and cyclic complement.

Let G be an abelian group and suppose that $A \subseteq \mathrm{Aut}(G)$. We say that A is cyclic subgroup regular(CSR) provided for all $1 \neq x \in G, 1 \neq \alpha \in A$, we have

$$\langle x\rangle^{\alpha} \cap \langle x\rangle = 1.$$

If A is a cyclic groupa and $A = \langle\alpha\rangle$, we say that α is a CSR automorphism.

If G is an abelian group and α is a CSR-automorphism of G, then the relative holomorph $G]\langle\alpha\rangle$ is a MCTI-group. In the last section we give numerical conditions for a finite, abelian group to have a CSR-automorphism. These conditions are related to the Singer Cycle of an elementary abelian p-group. Note that $Z_{10} \times Z_{10}$ has a CSR-automorphism, but $Z_{10} \times Z_{10} \times Z_{10}$ does not.

Structure of Some Weyl Modules

Elizabeth Wiggins

University of Florida

Abstract

Let G be a simple algebraic group over an algebraically closed field of characteristic $p > 0$. In this talk, we will examine groups of type B_4 and D_4, which are the classical groups SO(9) and SO(8), respectively. We will determine the structure of Weyl modules and characters of simple modules for some particular weights. We will also discuss an application in the theory of spherical buildings.

Some Properties of the Classical Numerical Range of a Matrix in $\mathfrak{sp}(n, \mathbb{C})$

Wen Yan and Jicheng Tao

Tuskegee University and China Jiliang University

Abstract

In this paper we obtained some relationships between the classical numerical range of a matrix $A \in \mathfrak{sp}(n, \mathbb{C})$ and the singular values of one of its submatrices. We also considered the circularity properties of the numerical range of some matrices in $\mathfrak{sp}(n, \mathbb{C})$ and obtain a necessary and sufficient condition for the numerical range to be circular when $n = 2$.