**Memorial University of Newfoundland** **Fall Semester 2025**
**Yorck Sommerhäuser** **MATH 3370: Sheet 6**

# Introductory Number Theory

**Problem 1:** Show that every Carmichael number is squarefree. (25 points)
(This is part of Korselt's theorem, which of course you cannot just cite for this purpose.)

**Problem 2:** If $p$ is prime, $p \equiv 3 \pmod 4$ and $a^2 + b^2 \equiv 0 \pmod p$, prove that $a \equiv b \equiv 0 \pmod p$. (25 points)
(This is Problem 16 for Chapter 4 in the course notes.)

**Problem 3:** Prove that $a^2 - 17b^2 = 14$ has no integer solutions. (25 points)
(This is Problem 17 for Chapter 4 in the course notes. The equation is a variant of the so-called Pell equation.)

**Problem 4:** The converse of Wilson's theorem states that if $(n-1)! \equiv -1 \pmod n$, then $n$ is prime. Show the following more precise statement, which extends Wilson's theorem:

1. $(n-1)! \equiv -1 \pmod n$ if $n$ is prime.
2. $(n-1)! \equiv 2 \pmod n$ if $n = 4$.
3. $(n-1)! \equiv 0 \pmod n$ in all other cases. (25 points)

(This is a variant of Problem 20 for Chapter 4 in the course notes.)

Due date: Monday, November 3, 2025. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.