Memorial University of Newfoundland **Fall Semester 2024**
Yorck Sommerhäuser **MATH 3370: Sheet 5**

Introductory Number Theory

Problem 1: Show that 567 has a multiplicative inverse modulo 893, and find this inverse. (25 points)
(This is a variant of Problem 5 for Chapter 4 in the course notes.)

Problem 2: Prove that $n^{20} - a^{20}$ is divisible by 165 if n and a are relatively prime to 165. (25 points)
(This is Problem 8 for Chapter 4 in the course notes.)

Problem 3: Let $f(x) = 375x^5 - 131x^4 + 15x^2 - 435x - 2$. Find the remainder when $f(97)$ is divided by 11. (25 points)
(This is Problem 3 for Chapter 4 in the course notes. The answer is indeed at the end of the notes, but, as always, you are supposed to justify your answer.)

Problem 4: Suppose that p is an odd prime. Show that

1. $1^{p-1} + 2^{p-1} + \ldots + (p-1)^{p-1} \equiv -1 \pmod{p}$. (13 points)
2. $1^p + 2^p + \ldots + (p-1)^p \equiv 0 \pmod{p}$. (12 points)

(This is Problem 14 for Chapter 4 in the course notes.)

Due date: Monday, October 21, 2024. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.