## Memorial University of Newfoundland Yorck Sommerhäuser

## Fall Semester 2024 MATH 3370: Sheet 10

## Introductory Number Theory

**Problem 1:** Suppose that (a, b, c) is a Pythagorean triple, i.e., a triple of positive integers satisfying  $a^2+b^2=c^2$ . Show that one of the integers a, b, or c is divisible by 5. (25 points)

**Problem 2:** Suppose that (a, b, c) is a Pythagorean triple. Show that the product ab is divisible by 12. (24 points)

**Problem 3:** Suppose that (a, b, c) is a Pythagorean triple with the property that c = b + 1.

- 1. Show that (a, b, c) is primitive. (4 points)
- 2. Show that there exists a positive integer v so that

$$a = 2v + 1$$
  $b = 2v(v + 1)$   $c = 2v(v + 1) + 1$ 

(20 points)

**Problem 4:** Suppose that a, b, and c are integers satisfying  $a^3 + b^3 + c^3 = 0$ .

- 1. Show that  $(a + b + c)^3 = 3(a + b)(a + c)(b + c)$ . (9 points)
- 2. Show that a, b, or c must be divisible by 3. (9 points)
- 3. Deduce the so-called first case of Fermat's last theorem for the exponent n = 3: If x, y, and z are integers satisfying  $x^3 + y^3 = z^3$ , then 3 divides xyz. (9 points)

Due date: Monday, November 25, 2024. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.