Memorial University of Newfoundland **Fall Semester 2021**
Yorck Sommerhäuser **MATH 3370: Sheet 3**

Introductory Number Theory

Problem 1:

1. Show that any number of the form $2^{4n+2}+1$ can be easily factored into two factors by using the identity

$$4x^4+1=(2x^2+2x+1)(2x^2-2x+1)$$

(12 points)

2. Using this identity, factor $2^{18}+1$ completely as a product of prime numbers. Do not use a calculator and write down all your computations in complete detail, as always. To justify that a specific number is prime, you may use the table at the end of the course notes. (13 points)

(This is Problem 2 for Chapter 3 in the course notes.)

Problem 2: Let a and b be positive integers. We have seen in class, and it is also shown in Theorem 2.9 on page 37 in the course notes, that

$$[a,b](a,b)=ab$$

Give a different proof of this result using the canonical decomposition of the numbers. (25 points)
(This is Problem 4 (c) for Chapter 3 in the course notes. A hint can be found on page 47 of the notes.)

Problem 3: A man sold his sheep for \$180 each and his cows for \$290 each. He received a total of \$2890. How many cows did he sell? (25 points)
(This is Problem 39 for Chapter 2 in the course notes.)

Problem 4:

1. Using Part 1 of Problem 1 on Sheet 1, show that 10^n+1 is divisible by 11 if n is odd. (8 points)

2. Using a similar reasoning, show that 10^n-1 is divisible by 11 if n is even. (8 points, continued on back)

3. Prove that an integer is divisible by 11 if and only if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is divisible by 11. (8 points)

4. Use the last statement to determine whether or not 85976 is divisible by 11. (1 point)

(This is a variant of Problem 11 for Chapter 3 in the course notes.)

Due date: Monday, October 4, 2021. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.