Memorial University of Newfoundland **Fall Semester 2021**
Yorck Sommerhäuser **MATH 3370: Sheet 2**

Introductory Number Theory

Problem 1: For $a = 4409$ and $b = 519$, find the greatest common divisor and find x and y such that $ax + by = \mathrm{gcd(a,b)}$ by making a four-column table with entries c_n, d_n, r_n, and q_n as on page 33 of the course notes. (25 points)
(This is Problem 11 (c) for Chapter 2 in the course notes. Similar questions are not unlikely to be on the midterm exam.)

Problem 2: For any integer k, show that the numbers $3k + 2$ and $5k + 3$ are relatively prime. (25 points)
(This is Problem 9 for Chapter 2 in the course notes.)

Problem 3: Let $\{f_n\}$ be the Fibonacci sequence, defined in Chapter 1 of the notes as well as in Problem 4 on Sheet 1. Prove that in finding (f_{n+2}, f_{n+1}) by the Euclidean algorithm, n divisions are necessary. (25 points)
(This is Problem 15 for Chapter 2 in the course notes.)

Problem 4:

1. Let a be a positive integer different from 1, and s and t positive integers. Prove that $(a^t - 1)$ divides $(a^s - 1)$ if and only if t divides s. (15 points)
2. Prove that $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$ for any positive integer a with $a \neq 1$. (10 points)

(This is Problem 16 for Chapter 2 in the course notes. At the end, the notes contain a hint for this problem.)

Due date: Monday, September 27, 2021. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.