Memorial University of Newfoundland
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Fall Semester 2020
MATH 3370: Sheet 9

Introductory Number Theory

Problem 1: Suppose that (a, b, c) is a Pythagorean triple, i.e., a triple of positive integers satisfying $a^2+b^2 = c^2$. Show that one of the integers a, b, or c is divisible by 5. (25 points)

Problem 2: Suppose that (a, b, c) is a Pythagorean triple. Show that the product ab is divisible by 12. (24 points)

Problem 3: Suppose that a, b, and c are integers satisfying $a^3 + b^3 + c^3 = 0$.

1. Show that $(a + b + c)^3 = 3(a + b)(a + c)(b + c)$. (9 points)
2. Show that a, b, or c must be divisible by 3. (9 points)
3. Deduce the so-called first case of Fermat's last theorem for the exponent $n = 3$: If x, y, and z are integers satisfying $x^3 + y^3 = z^3$, then 3 divides xyz. (9 points)

Problem 4: In the ring **G** of Gaussian integers, determine which of the following divisibility relations hold.

1. $(2 + 3i) \mid (5 - i)$ (8 points)
2. $3 \mid (2 - i)(3 + i)$ (8 points)
3. $(3 - 2i) \mid 26$ (8 points)

Due date: Monday, November 16, 2020. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.