**Memorial University of Newfoundland** **Fall Semester 2020**
**Yorck Sommerhäuser** **MATH 3370: Sheet 6**

# Introductory Number Theory

**Problem 1:** Show that every Carmichael number is squarefree. (25 points)
(This is part of Korselt's theorem from Lecture 16, which of course you cannot just cite for this purpose.)

**Problem 2:** If $p$ is prime, $p \equiv 3 \pmod 4$ and $a^2 + b^2 \equiv 0 \pmod p$, prove that $a \equiv b \equiv 0 \pmod p$. (25 points)

**Problem 3:** Prove that $a^2 - 17b^2 = 14$ has no integer solutions. (25 points)

**Problem 4:** The converse of Wilson's theorem states that if $(n-1)! \equiv -1 \pmod n$, then $n$ is prime. Show the following more precise statement, which extends Wilson's theorem:

1. $(n-1)! \equiv -1 \pmod n$ if $n$ is prime.
2. $(n-1)! \equiv 2 \pmod n$ if $n = 4$.
3. $(n-1)! \equiv 0 \pmod n$ in all other cases. (25 points)

Due date: Monday, October 26, 2020. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.