

Introductory Number Theory

Problem 1: Show that 567 has a multiplicative inverse modulo 893, and find this inverse. (25 points)

Problem 2: Prove that $n^{20} - a^{20}$ is divisible by 165 if n and a are relatively prime to 165. (25 points)

Problem 3: Let $f(x) = 375x^5 - 131x^4 + 15x^2 - 435x - 2$. Find the remainder when $f(97)$ is divided by 11. (25 points)

Problem 4: Suppose that p is an odd prime. Show that

1. $1^{p-1} + 2^{p-1} + \dots + (p-1)^{p-1} \equiv -1 \pmod{p}$. (13 points)

2. $1^p + 2^p + \dots + (p-1)^p \equiv 0 \pmod{p}$. (12 points)

Due date: Monday, October 19, 2020. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write your student number on your solution.