Memorial University of Newfoundland **Fall Semester 2020**
Yorck Sommerhäuser **MATH 3370: Sheet 4**

Introductory Number Theory

Problem 1: The last digits in the decimal expansion of

$$n := 100! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot 100$$

are zeroes. How many zeroes are there? (25 points)

Problem 2: Given the number 2492, double the units digit and subtract it from the number formed by the other digits. We get $249 - 2 \times 2 = 245$. Repeating this algorithm we get $24 - 2 \times 5 = 14$. Since 14 is clearly divisible by 7, the original number 2492 must be divisible by 7. Prove this rule for checking divisibility by 7. (25 points)

Problem 3: Let p denote an odd prime. As discussed in Lecture 11, it is conjectured that there are infinitely many twin primes $p, p+2$. Prove that the only prime triple $p, p+2, p+4$ is the triple 3, 5, 7. (25 points)

Problem 4: Dirichlet proved that there are always infinitely many primes in any arithmetic sequence $a, a+d, a+2d, \ldots$, where $(a, d) = 1$. Prove that there are infinitely many primes in the arithmetic sequence $3, 7, 11, 15, \ldots$.
(Hint: Consider the numbers of the form $4p_1p_2 \ldots p_r - 1$, where each p_i is prime and of the form $4k - 1$.) (25 points)

Due date: Monday, October 12, 2020. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.