Memorial University of Newfoundland **Fall Semester 2020**
Yorck Sommerhäuser **MATH 3370: Sheet 3**

Introductory Number Theory

Problem 1:

1. Show that any number of the form $2^{4n+2}+1$ can be easily factored into two factors by using the identity

$$4x^4+1=(2x^2+2x+1)(2x^2-2x+1)$$

(12 points)

2. Using this identity, factor $2^{18}+1$ completely as a product of prime numbers. Do not use a calculator and write down all your computations in complete detail, as always. To justify that a specific number is prime, you may use the provided table of primes. (13 points)

Problem 2: Let a and b be positive integers. We have seen in Theorem 2.9 of Lecture 9 that

$$[a,b](a,b)=ab$$

Give a different proof of this result using the canonical decomposition of the numbers discussed at the end of Lecture 9 (see also Lecture 10). (25 points)

Problem 3: A man sold his sheep for \$180 each and his cows for \$290 each. He received a total of \$2890. How many cows did he sell? (25 points)

Problem 4:

1. Using Part 1 of Problem 1 on Sheet 1, show that 10^n+1 is divisible by 11 if n is odd. (8 points)

2. Using a similar reasoning, show that 10^n-1 is divisible by 11 if n is even. (8 points, continued on back)

3. Prove that an integer is divisible by 11 if and only if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is divisible by 11. (8 points)

4. Use the last statement to determine whether or not 85976 is divisible by 11. (1 point)

Due date: Monday, October 5, 2020. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.