Memorial University of Newfoundland **Fall Semester 2020**
Yorck Sommerhäuser **MATH 3370: Sheet 2**

Introductory Number Theory

Problem 1: For $a = 4409$ and $b = 519$, find the greatest common divisor and find x and y such that $ax + by = \gcd(a, b)$ by making a four-column table with entries c_n, d_n, r_n, and q_n as on page 4 of Lecture 6/7. (25 points)

Problem 2: For any integer k, show that the numbers $3k + 2$ and $5k + 3$ are relatively prime. (25 points)

Problem 3: Let $\{f_n\}$ be the Fibonacci sequence, defined in Lecture 1 as well as in Problem 4 on Sheet 1. Prove that in finding (f_{n+2}, f_{n+1}) by the Euclidean algorithm, n divisions are necessary. (25 points)

Problem 4:

1. Let a be a positive integer different from 1, and s and t positive integers. Prove that $(a^t - 1)$ divides $(a^s - 1)$ if and only if t divides s. (15 points)
2. Prove that $(a^m - 1, a^n - 1) = a^{(m,n)} - 1$ for any positive integer a with $a \neq 1$. (10 points)

Hint for Part 1 of Problem 4: If $s \geq t$, then there exist integers q and r with $q \geq 1$ and $0 \leq r < t$ such that $s = qt + r$. Then

$$\begin{aligned} \frac{a^s - 1}{a^t - 1} = \frac{a^{qt+r} - 1}{a^t - 1} &= \frac{a^r a^{qt} - a^r + a^r - 1}{a^t - 1} \\ &= a^r \left(\frac{(a^t)^q - 1}{a^t - 1} \right) + \frac{a^r - 1}{a^t - 1}. \end{aligned}$$

Due date: Monday, September 28, 2020. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.