Introductory Number Theory

Problem 1: For a Gaussian integer $\alpha \in \mathbf{G}$, show that α is a unit if and only if $N(\alpha) = 1$.   (25 points)
(This problem was already stated on page 2 of Lecture 27.)

Problem 2: For $\beta = 8 - 9i$ and $\alpha = 3 + 5i$, find γ and δ satisfying the division algorithm in the ring $\mathbf{G}$ of Gaussian integers.   (25 points)

Problem 3: In the ring $\mathbf{G}$ of Gaussian integers, find a greatest common divisor of $\alpha = 531 + 582i$ and $\beta = 54 - 101i$.   (25 points)
(This problem was already stated at the end of Lecture 28.)

Problem 4: Factor the Gaussian integer $304 + 228i$ completely into primes.
                                                         (26 points)

Due date: Monday, November 23, 2020. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.