

Introductory Number Theory

Problem 1: In the ring \mathbf{G} of Gaussian integers, determine which of the following divisibility relations hold.

1. $(2 + 3i) \mid (5 - i)$ (6 points)
2. $3 \mid (2 - i)(3 + i)$ (6 points)
3. $(3 - 2i) \mid 26$ (6 points)
4. $(4 - 5i) \mid (91 - 22i)$ (6 points)

(This is Problem 5 for Chapter 9 in the course notes.)

Problem 2: For $\beta = 8 - 9i$ and $\alpha = 3 + 5i$, find γ and δ satisfying the division algorithm in the ring \mathbf{G} of Gaussian integers. (25 points)

(This is Problem 3 for Chapter 9 in the course notes.)

Problem 3: In the ring \mathbf{G} of Gaussian integers, find a greatest common divisor of $\alpha = 531 + 582i$ and $\beta = 54 - 101i$. (25 points)

(This problem is taken from page 97 in Chapter 9 in the course notes.)

Problem 4: Factor the Gaussian integer $304 + 228i$ completely into primes. (26 points)

(This is part of Problem 9 for Chapter 9 in the course notes.)

Due date: There is no due date. The completion of this sheet is voluntary. The solutions will not be collected and will not be marked. However, these exercises provide valuable practice for the final exam, which takes place on Friday, December 7, from 9:00 am to 12:00 m, in our usual classroom.