Memorial University of Newfoundland **Winter Semester 2025**
Yorck Sommerhäuser **MATH 4321/6320: Sheet 5**

Group Theory

Problem 1: An abelian group is called indecomposable if it cannot be written as the internal direct product of two proper subgroups. Show that a cyclic group of prime power order is indecomposable. (20 points)

Problem 2: List all abelian groups of order 8281, up to isomorphism. (I.e., make a list of abelian groups of order 8281 so that no two groups on this list are isomorphic and every abelian group of order 8281 is isomorphic to a group on this list.) (30 points)

Problem 3:

1. In the symmetric group S_n on n letters, write a cycle $\sigma := (i_1, i_2, \ldots, i_m)$ as a product of $m-1$ transpositions. (18 points)
2. Show that the sign of σ is $\varepsilon(\sigma) = (-1)^{m-1}$. (2 points)

Problem 4:

1. Compute the order $|A_4|$ of the alternating group on four letters. (4 points)
2. On a separate letter-size page, make a list of all subgroups of A_4, with the trivial subgroup at the bottom and the entire group at the top. Draw an ascending arrow between subgroups if one subgroup contains the other, but there is no subgroup strictly in between. (Such a diagram is often called a Hasse diagram.) (18 points)
3. Determine which of these subgroups are normal and which are characteristic. (4 points)
4. Conclude that A_4 does not contain a subgroup of order 6. (4 points)

Due date: Tuesday, February 18, 2025. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.