Memorial University of Newfoundland **Winter Semester 2025**
Yorck Sommerhäuser **MATH 4321/6320: Sheet 2**

Group Theory

Problem 1: Consider the subset

$$V := \{\mathrm{id}, (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$$

of the symmetric group S_4 on four letters.

1. Make a multiplication table for V. (10 points)
2. Use this multiplication table to show that V is a subgroup of S_4. (5 points)
3. Show that V is a normal subgroup of S_4. (10 points)
   (Hint: The formula for the conjugate of a cycle that we did in class can be helpful here.)

Problem 2: Suppose that G is a group in which every element that is different from the unit element has order 2. Show that G is abelian. (15 points)

Problem 3: Suppose that G is an abelian group of order p^2, where p is a prime. Show that $G \cong C_{p^2}$ or $G \cong C_p \times C_p$. (40 points)

(Hint: In general, the Cartesian product $G \times H$ of two groups G and H is a group with respect to the operation

$$(g,h)(g',h') := (gg',hh')$$

You do not need to show that in writing, but you should verify this for yourself.)

Problem 4: Suppose that G is a group of order 4. Show that $G \cong C_4$ or $G \cong C_2 \times C_2$. (20 points)

Due date: Tuesday, January 28, 2025. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.