Memorial University of Newfoundland **Fall Semester 2019**
Yorck Sommerhäuser **MATH 4331/6328: Sheet 7**

Galois Theory

Problem 1:

1. Show that the multiplicative group $\mathbb{Q}^\times := \mathbb{Q} \setminus \{0\}$ of nonzero rational numbers is not cyclic. (15 points)
2. Suppose that K is a field of characteristic zero. Show that its multiplicative group $K^\times := K \setminus \{0\}$ is not cyclic. (10 points)

(This is the first part of Exercise 81 on page 70 in the textbook.)

Problem 2: Suppose that p is a prime and that K is an infinite field of characteristic $p > 0$.

1. Show that there is no algebraic element $\alpha \in K$ with the property that $K = \mathbb{Z}_p(\alpha)$, where $\mathbb{Z}_p$ is viewed as the prime field of K. (5 points)
2. Show that the multiplicative group $K^\times := K \setminus \{0\}$ is not cyclic. (20 points)

(This is the second part of Exercise 81 on page 70 in the textbook, which contains a hint for the problem.)

Problem 3: Let K be a field with eight elements.

1. Copy down the addition table for K that you have obtained in the first part of Problem 4 on Sheet 5 again. (2 points)
2. Make an addition table for the group $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. (8 points)
3. Explain how the two tables are related. (15 points)

Problem 4: Let K be a field with eight elements.

1. Copy down the multiplication table for K that you have obtained in the second part of Problem 4 on Sheet 5 again. (2 points)
2. Make an addition table for the group $\mathbb{Z}_7$. (8 points)
3. Explain how the two tables are related. (15 points)

(You do not need to copy the old tables by hand; you can also submit photocopies. Furthermore, you can make corrections if there were mistakes.)

Due date: Wednesday, November 13, 2019.