Memorial University of Newfoundland **Fall Semester 2019**
Yorck Sommerhäuser **MATH 4331/6328: Sheet 5**

Galois Theory

Problem 1: Suppose that $F \subset E$ is a field extension.

1. Show that the set
$$K := \{\alpha \in E \mid \alpha \text{ is algebraic over } F\}$$
is a subfield of E. (22 points)
2. Show that $F \subset K$. (3 points)

(This is Exercise 72 (ii) on page 58 in the textbook, which contains a hint for the problem. The field K is called the algebraic closure of F in E.)

Problem 2: Consider the field $K := \mathbb{Z}_p(t)$ of rational functions with coefficients in $\mathbb{Z}_p$. Show that the polynomial
$$f(x) := x^p - t \in K[x]$$
is irreducible. (25 points)

(This is Exercise 75 on page 58 in the textbook, which contains a hint for the problem.)

Problem 3: Show that the polynomial
$$f(x) = x^4 - 10x^2 + 1 \in \mathbb{Q}[x]$$
is irreducible. (25 points)

(This is Exercise 67 on page 43 in the textbook, which contains a hint for the problem.)

Problem 4: By Moore's theorem (Corollary 53 on page 57 in the textbook), a field K that contains 8 elements is uniquely determined up to isomorphism.

1. Write out an addition table for K. (13 points)
2. Write out a multiplication table for K. (12 points)

(This is Exercise 61 on page 38 in the textbook, which contains a hint for the problem.)

Due date: Monday, October 28, 2019.