Memorial University of Newfoundland **Winter Semester 2022**
Yorck Sommerhäuser **MATH 4310: Sheet 4**

Complex Function Theory

Problem 1: For the curve

$$\gamma : [0, 2\pi] \to \mathbb{C},\ t \mapsto e^{2it} + 2e^{-it}$$

find the winding number $W(\gamma)$ defined in Problem 1 on Sheet 3.

(Hint: Use the formula $W(\delta_1\delta_2) = W(\delta_1) + W(\delta_2)$ proved in class and Problem 2 on Sheet 3. Do not use the argument principle for this problem.) (25 points)

Problem 2: Consider the meromorphic function $f(z) = \frac{z^3+2}{z}$.

1. Find all zeros and poles of f inside the unit disk $D := \{z \in \mathbb{C} \mid |z| < 1\}$. (15 points)
2. Explain how the first part and the argument principle can be used to rederive the result of Problem 1. (10 points)

(Hint: Note that $\gamma(t) = f(e^{it})$.)

Problem 3: Show that the polynomial $p(z) := z^6 - 4z + 2$ has exactly one root inside the unit disk.

(Hint: Use Rouché's theorem.) (25 points)

Problem 4: Find the inverse Laplace transform of the function

$$F(s) := \frac{s^2 - 1}{(s^2 + 1)^2}$$

(Hint: Proceed as in Example 1 of Section 82.) (25 points)

Due date: Monday, February 14, 2022. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.