Memorial University of Newfoundland **Winter Semester 2022**
Yorck Sommerhäuser **MATH 4310: Sheet 2**

Complex Function Theory

Problem 1: For nonnegative real numbers a and b, show that

$$\int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2} dx = \frac{\pi}{2}(b - a)$$

(Hint: Proceed as in the example given in Section 75 of the textbook; in particular, use the contour shown in Figure 97 on page 269.) (25 points)

Problem 2: For a real number a with $-1 < a < 3$, show that

$$\int_0^\infty \frac{x^a}{(x^2+1)^2} dx = \frac{(1-a)\pi}{4\cos(a\pi/2)}$$

(Hint: Proceed as in Section 76 of the textbook; in particular, use the same contour as in the preceding problem. Compare also Section 77 of the textbook. Note that, for $a = 0$, we recover the result in Problem 2 and Problem 3 on Sheet 1.) (25 points)

Problem 3: By using contour integration with the unit circle as contour, show that

$$\int_{-\pi}^{\pi} \sin^4(\theta) d\theta = \frac{3\pi}{4}$$

(Hint: Proceed as in Section 78 of the textbook.) (25 points)

Problem 4:

1. Find the indefinite integral $\int \sin^4(\theta) d\theta$. (20 points)
2. Use the result of the first part to prove again that

$$\int_{-\pi}^{\pi} \sin^4(\theta) d\theta = \frac{3\pi}{4}$$

(5 points)

Due date: Monday, January 31, 2022. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.