Memorial University of Newfoundland **Winter Semester 2022**
Yorck Sommerhäuser **MATH 4310: Sheet 1**

Complex Function Theory

Problem 1: For a complex number $z = x + iy$, show that

$$|\sin z|^2 = \sin^2 x + \sinh^2 y$$

(Hint: This is part of Exercise 9 for Section 33 in the textbook, which contains a hint for the problem. The definitions of sine and hyperbolic sine for complex arguments is also given in that section.) (25 points)

Problem 2: Using the same contour as in Section 72 of the textbook, show that

$$\int_0^\infty \frac{1}{(x^2+1)^2}\,dx = \frac{\pi}{4}$$

(Remark: In particular, you are supposed to use contour integration in this problem. Note that the lower boundary of the integral is 0, not $-\infty$.) (25 points)

Problem 3:

1. Using the trigonometric substitution $x = \tan(\theta)$, find the indefinite integral

   $$\int \frac{1}{(x^2+1)^2}\,dx$$

   (20 points)

2. Use the result of the first part to prove again that

   $$\int_0^\infty \frac{1}{(x^2+1)^2}\,dx = \frac{\pi}{4}$$

   (5 points)

(Remark: Using the method of partial fractions, it can be shown that any rational function can be integrated in terms of elementary functions. The integral in the first part is one of the necessary integrals.)

Problem 4: Using the same contour as in Section 72 of the textbook and in Problem 2, show that

$$\int_0^\infty \frac{x\sin(2x)}{x^2+3}dx = \frac{\pi}{2}e^{-2\sqrt{3}}$$

(Hint: Use Jordan's lemma.) (25 points)

Due date: Monday, January 24, 2022. Write your solution on letter-sized paper and send your solution back to me via e-mail. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to write down your student number on your solution.