**Memorial University of Newfoundland** **Winter Semester 2021**
**Yorck Sommerhäuser** **MATH 4230: Sheet 8**

# Differential Geometry

**Problem 1:** Consider the parametrisation $\alpha$ of the intersection of the cylinder $S$ with the plane $P$ that was considered in Problem 1 of Sheet 5. Find the covariant derivative $\frac{D\alpha'(t)}{dt}$ of $\alpha'$ along $\alpha$. (25 points)

**Problem 2:** On the unit sphere

$$S := S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$$

consider the curve $\alpha : [0, \frac{\pi}{2}] \to S$ given by

$$\alpha(t) = (\cos(t), 0, \sin(t))$$

that begins at $p := \alpha(0) = (1, 0, 0)$ and ends at $q := \alpha(\pi/2) = (0, 0, 1)$.

1. For the vector $v_1 = (0, 0, 1) \in T_pS$, find the vector $v_2 \in T_qS$ that arises by transporting $v_1$ parallelly along $\alpha$. (13 points)
2. For the vector $w_1 = (0, 1, 0) \in T_pS$, find the vector $w_2 \in T_qS$ that arises by transporting $w_1$ parallelly along $\alpha$. (12 points)

(Hint: Find suitable parallel vector fields along $\alpha$.)

**Problem 3:** Suppose that $S$ is a regular oriented surface, that $\alpha : (-\varepsilon, \varepsilon) \to S$ is a differentiable curve, and that $v(t)$ and $w(t)$ are vector fields along $\alpha$. Show that

$$\frac{d}{dt}\langle v(t), w(t)\rangle = \langle \frac{Dv(t)}{dt}, w(t)\rangle + \langle v(t), \frac{Dw(t)}{dt}\rangle$$

(25 points)

**Problem 4:** Suppose that $f : U \to \mathbb{R}^3$ is a parametrised surface defined on the open set $U \subset \mathbb{R}^2$, and that $M \in SO(3)$ is an orthogonal matrix with determinant 1. Define the parametrised surface

$$g(u, v) := Mf(u, v) + w$$

where $w \in \mathbb{R}^3$ is a fixed vector. Show that the surfaces $f$ and $g$ have the same first fundamental form and the same second fundamental form. (25 points)

(Remark: The more difficult converse of this statement is part of the fundamental theorem of the local theory of surfaces in Section 4-3 on page 239. Remember that a similar statement about curves appeared in Problem 4 on Sheet 1.)

Due date: Thursday, March 25, 2021. Write your solution on letter-sized paper, scan it and submit it to the assignment box for Sheet 8 in the Brightspace site for this course. Begin your solution with a cover sheet. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Similarly, prove every assertion that you make in full detail. It is not necessary to copy down the problems again.