Memorial University of Newfoundland
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Winter Semester 2024
MATH 3210: Sheet 9

Introduction to Complex Analysis

Problem 1: If C is a circle of radius 3 around the origin, find

$$\int_C \frac{e^{-2z^2}}{(z-2i)(z-5)} dz$$

Problem 2: A holomorphic function that is defined in the entire Argand plane is called an entire function. Show that there is no nonzero entire function f that satisfies $f(0) = 0$ and $|f(z)| \leq 1$ for all $z \in \mathbb{C}$ with $|z| \geq 1$.

Problem 3: Suppose that

$$P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$$

is a polynomial of degree n. Show that for every $\varepsilon \in \mathbb{R}$ with $0 < \varepsilon < 1$, there exists $R > 0$ such that

$$(1-\varepsilon)|a_n||z^n| \leq |P(z)| \leq (1+\varepsilon)|a_n||z^n|$$

for all $z \in \mathbb{C}$ satisfying $|z| > R$.

Problem 4: If

$$z_n := \frac{\cos(n)}{n^2} + i\frac{2-n^2}{1+2n^2}$$

decide whether the sequence (z_n) converges. If it converges, find its limit.

Due date: There is no due date. The completion of this sheet is voluntary. The solutions will not be collected and will not be marked. However, these exercises can be used as practice for the final exam.