Memorial University of Newfoundland **Winter Semester 2024**
Yorck Sommerhäuser **MATH 3210: Sheet 7**

Introduction to Complex Analysis

Problem 1: Suppose that

$$w(t) = u(t) + iv(t) \qquad \text{and} \qquad W(t) = U(t) + iV(t)$$

are two complex-valued function of a real variable $t \in (a, b)$.

1. Prove the product rule
$$(wW)'(t) = w'(t)W(t) + w(t)W'(t)$$
(22 points)

2. Explain how the product rule can be used to derive Equation (3) on page 112 in Section 36 of the textbook. (3 points)

Problem 2: For the function $f(z) = z^2$, compute the contour integrals

$$\int_{C_1} f(z)dz \qquad \text{and} \qquad \int_{C_2} f(z)dz$$

where C_1 is the curve parametrized by $\gamma_1(t) = t + it$ and C_2 is the curve parametrized by $\gamma_2(t) = t^2 + it^2$, where in both cases $0 \leq t \leq 1$. Draw a picture of both C_1 and C_2 in the Argand plane. Discuss in detail how the integrals are related: If they are equal, find a reason why they are equal; if they are different, explain how their difference comes about. (25 points)

Problem 3: For the function $f(z) = \frac{z+z^2}{z-2}$, compute the contour integrals

$$\int_{C_1} f(z)dz \qquad \text{and} \qquad \int_{C_2} f(z)dz$$

where C_1 is the curve parametrized by $\gamma_1(t) = 2 + 5e^{it}$ and C_2 is the curve parametrized by $\gamma_2(t) = 2 + 5e^{2it}$, where in both cases $0 \leq t \leq 2\pi$. Draw a picture of both C_1 and C_2 in the Argand plane. Discuss in detail how the integrals are related: If they are equal, find a reason why they are equal; if they are different, explain how their difference comes about. (25 points)

Problem 4: For the function $f(z) = \bar{z} - 1$, compute the contour integrals

$$\int_{C_1} f(z)dz \qquad \text{and} \qquad \int_{C_2} f(z)dz$$

where C_1 is the curve parametrized by $\gamma_1(t) = 1 + \cos(t) + i\sin(t)$ and C_2 is the curve parametrized by $\gamma_2(t) = 1 + \cos(t) - i\sin(t)$, where in both cases $0 \leq t \leq 2\pi$. Draw a picture of both C_1 and C_2 in the Argand plane. Discuss in detail how the integrals are related: If they are equal, find a reason why they are equal; if they are different, explain how their difference comes about. (25 points)

Due date: Monday, March 18, 2024. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.