Memorial University of Newfoundland **Winter Semester 2024**
Yorck Sommerhäuser **MATH 3210: Sheet 6**

Introduction to Complex Analysis

Problem 1: Let $f(z) = u(r, \theta) + iv(r, \theta)$, where $z = re^{i\theta}$, be a function that is infinitely often complex differentiable for all complex numbers z in a domain D. Show that

$$r^2 u_{rr}(r, \theta) + r u_r(r, \theta) + u_{\theta\theta}(r, \theta) = 0$$

and

$$r^2 v_{rr}(r, \theta) + r v_r(r, \theta) + v_{\theta\theta}(r, \theta) = 0$$

(Hint: Use the Cauchy-Riemann equations in polar form (Sec. 22, Eq. (6), p. 66). Note that we used the notation $\tilde{u}$ and $\tilde{v}$ instead of u and v in class. The equation above is called the polar form of the Laplace equation.) (25 points)

Problem 2: Find i^i. Give both all possible values and the principal value. (25 points)

Problem 3: If $z = (x, y) = x + iy$, show that

1. $|\sin(z)|^2 = \sin^2(x) + \sinh^2(y)$ (13 points)
2. $|\cos(z)|^2 = \cos^2(x) + \sinh^2(y)$ (12 points)

Problem 4: Show that the trigonometric functions $\cot(z)$ and $\csc(z)$ are analytic where they are defined, i.e., when $z \neq n\pi$ for $n \in \mathbb{Z}$, and that

$$\frac{d}{dz} \cot(z) = -\csc^2(z) \quad \text{and} \quad \frac{d}{dz} \csc(z) = -\csc(z)\cot(z)$$

(25 points)

Due date: Monday, March 11, 2024. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.