Memorial University of Newfoundland **Winter Semester 2024**
Yorck Sommerhäuser **MATH 3210: Sheet 1**

Introduction to Complex Analysis

Problem 1: Show that multiplication of complex numbers is associative. In other words, show that for three complex numbers $z_1 = (x_1, y_1)$, $z_2 = (x_2, y_2)$, and $z_3 = (x_3, y_3)$, we have

$$((x_1, y_1)(x_2, y_2))(x_3, y_3) = (x_1, y_1)((x_2, y_2)(x_3, y_3))$$

or $(z_1 z_2)z_3 = z_1(z_2 z_3)$. (25 points)

Problem 2: Show that multiplication of complex numbers is distributive. In other words, using the notation of Problem 1, show that

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$$

(25 points)

Problem 3: Compute explicitly the complex number

$$\frac{3 - 2i}{4 - 3i} \cdot \frac{2 + i}{1 + 2i}$$

and find its real and imaginary parts. (25 points)

Problem 4: In the Argand plane, draw a picture of all complex numbers that satisfy the equations $|z - 2 + i| = 5$. Afterwards, do the same for the equation $|z - 2 + i| = |z + 2 - 3i|$. Explain how these pictures come about. (25 points)

Due date: Monday, January 22, 2024. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.