Memorial University of Newfoundland **Winter Semester 2024**
Yorck Sommerhäuser **MATH 3210: Sheet 10**

Introduction to Complex Analysis

Problem 1: Decide whether the series $\sum_{n=0}^{\infty} e^{in}$ converges. If it converges, find its sum, i.e., its limit.

Problem 2: For the function

$$f(z) = \frac{z+5}{z^2+z-2}$$

find the Maclaurin series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

Make a table with the explicit values a_0, a_1, a_2, a_3, a_4, and a_5, and determine the radius of convergence of the Maclaurin series.
(Hint: Use partial fraction decomposition and the fact, which we have not yet shown in class, that a power series representation is unique, so that every power series representation must be the Maclaurin series. It is possible, but extremely tedious, to use the explicit formulas for the coefficients of the Maclaurin series in this example. This remark also applies to the next problem.)

Problem 3: For the function

$$f(z) = \frac{1}{z^2-2z-24}$$

find the Taylor series

$$f(z) = \sum_{n=0}^{\infty} a_n (z-1)^n$$

about the point 1. Make a table with the explicit values a_0, a_1, a_2, a_3, a_4, a_5, and a_{1001}, and determine the radius of convergence of the Taylor series.

Problem 4: For the function

$$f(z) = \frac{1}{z^2 - 2z - 24}$$

find the Laurent series

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-6)^n$$

about the point 6 that converges for points close to 6. Make a table with the explicit values c_{-3}, c_{-2}, c_{-1}, c_0, c_1, c_2, c_3, and determine where precisely the Laurent series converges.

Due date: There is no due date. The completion of this sheet is voluntary. The solutions will not be collected and will not be marked. However, these exercises can be used as practice for the final exam.