

Introduction to Complex Analysis

Problem 1: If

$$z_n := \frac{\cos(n)}{n^2} + i \frac{2 - n^2}{1 + 2n^2}$$

decide whether the sequence (z_n) converges. If it converges, find its limit.

(25 points)

Problem 2: Decide whether the series $\sum_{n=0}^{\infty} e^{in}$ converges. If it converges, find its sum, i.e., its limit.

(25 points)

Problem 3: For the function

$$f(z) = \frac{z + 5}{z^2 + z - 2}$$

find the Maclaurin series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

Make a table with the explicit values a_0, a_1, a_2, a_3, a_4 , and a_5 , and determine the radius of convergence of the Maclaurin series.

(Hint: Use partial fraction decomposition and the fact, which we have not yet shown in class, that a power series representation is unique, so that every power series representation must be the Maclaurin series. It is possible, but extremely tedious, to use the explicit formulas for the coefficients of the Maclaurin series in this example. This remark also applies to the next problem.)

(25 points)

Problem 4: For the function

$$f(z) = \frac{1}{z^2 - 2z - 24}$$

find the Taylor series

$$f(z) = \sum_{n=0}^{\infty} a_n (z - 1)^n$$

about the point 1. Make a table with the explicit values $a_0, a_1, a_2, a_3, a_4, a_5$, and a_{1001} , and determine the radius of convergence of the Taylor series.

(25 points)

Due date: Tuesday, November 20, 2018. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.