Memorial University of Newfoundland **Fall Semester 2018**
Yorck Sommerhäuser **MATH 3210: Sheet 8**

Introduction to Complex Analysis

Problem 1: For $z \in \mathbb{C}$, we define the complex cosine and sine functions as

$$\cos(z) := \frac{e^{iz} + e^{-iz}}{2} \qquad \sin(z) := \frac{e^{iz} - e^{-iz}}{2i}$$

1. Show that for a real number $z \in \mathbb{R}$, this definition agrees with the usual definition of cosine and sine. (8 points)
2. Show that $\cos(z_1 + z_2) = \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)$. (8 points)
3. Show that $\sin(z_1 + z_2) = \sin(z_1)\cos(z_2) + \cos(z_1)\sin(z_2)$. (8 points)

Problem 2: If C is a circle of radius 3 around the origin, find

$$\int_C \frac{e^{-2z^2}}{(z-2i)(z-5)} dz$$

(25 points)

Problem 3: A holomorphic function that is defined in the entire Argand plane is called an entire function. Show that there is no nonzero entire function f that satisfies $f(0) = 0$ and $|f(z)| \leq 1$ for all $z \in \mathbb{C}$ with $|z| \geq 1$. (25 points)

Problem 4: Suppose that

$$P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$$

is a polynomial of degree n. Show that for every $\varepsilon \in \mathbb{R}$ with $0 < \varepsilon < 1$, there exists $R > 0$ such that

$$(1 - \varepsilon)|a_n||z^n| \leq |P(z)| \leq (1 + \varepsilon)|a_n||z^n|$$

for all $z \in \mathbb{C}$ satisfying $|z| > R$. (26 points)

Due date: Tuesday, November 13, 2018. Please write your solution on letter-sized paper, and write your name on your solution. Please give all computations in full detail, and explain your computations in English, using complete sentences. It is not necessary to copy down the problems again or to submit this sheet with your solution.