**Memorial University of Newfoundland** **Fall Semester 2018**
**Yorck Sommerhäuser** **MATH 3210: Sheet 3**

# Introduction to Complex Analysis

**Problem 1:** Find

$$\lim_{z\to i}\frac{z-1}{z+1} \qquad \lim_{z\to\infty}\frac{z^2-i}{z^2+i} \qquad \lim_{z\to\infty}\frac{z^3-i}{z^2+i} \qquad \lim_{z\to 3i}\frac{z^2+4z+4}{z^2+2z-3iz-6i}$$

(As always, fully justify your answer.) (6 points each)

**Problem 2:** If $\lim_{z\to z_0} f(z) = w_0$, show that $\lim_{z\to z_0} |f(z)| = |w_0|$.
(Hint: Use the $\varepsilon$–$\delta$–definition that appears in Section 14 on page 43 of the textbook. The reverse triangle inequality in Section 4 on page 10 can help.) (25 points)

**Problem 3:** Suppose that $\lim_{z\to z_0} f(z) = 0$ and that the function $g(z)$ is bounded in a neighbourhood $U$ of $z_0$ in the sense that there is a constant $M$ such that $|g(z)| \leq M$ for all $z \in U$. Using the $\varepsilon$–$\delta$–definition of a limit, show that $\lim_{z\to z_0} f(z)g(z) = 0$. (26 points)

**Problem 4:** Show that $\lim_{z\to 0}\frac{z^3}{\overline{z}^3}$ does not exist. (25 points)

Due date: Tuesday, October 2, 2018. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.