Memorial University of Newfoundland **Fall Semester 2023**
Yorck Sommerhäuser **MATH 3320: Sheet 3**

Abstract Algebra

Problem 1: Suppose that $G = \langle g \rangle$ is a cyclic group of order 18 with generator g. Find all subgroups of G and draw the Hasse diagram of G. (25 points)

Problem 2: Suppose that p and q are distinct prime numbers. Suppose further that g and h are two commuting elements in a group G of order p and q, respectively. Show that the order of gh is pq.

(Remark: That g and h commute means that $gh = hg$.) (25 points)

Problem 3: Suppose that $G = \{e_G, g, g^2\}$ is a cyclic group of order 3 and that $H = \{e_H, h\}$ is a cyclic group of order 2. Show that cartesian product $G \times H$ is cyclic of order 6, and find a generator.

(Hint: See Problem 1 on Sheet 1 and Theorem 2 in Section 2.2 of the textbook.) (25 points)

Problem 4: Suppose that G is a finite group whose only subgroups are G itself and $\{e\}$, the subgroup consisting only of the neutral element. If G does not consist only of the unit element, show that G is cyclic and that the order $|G|$ is prime. (25 points)

Due date: Monday, October 2, 2023. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.