Memorial University of Newfoundland **Fall Semester 2023**
Yorck Sommerhäuser **MATH 3320: Sheet 2**

Abstract Algebra

Problem 1: Let $X := \{1, \dots, n\}$ be the set consisting of the first n integers. For distinct elements $k_1, k_2, \dots, k_n \in X$, we use the notation

$$\tau = (k_1, k_2, \dots, k_m)$$

for the map $\tau\colon X \to X$ with $\tau(k_1) = k_2$, $\tau(k_2) = k_3, \dots, \tau(k_{m-1}) = k_m$, and $\tau(k_m) = k_1$, while all other elements $i \in X$ are fixed in the sense that $\tau(i) = i$. τ is called a cycle of length m. For a bijective map $\sigma\colon X \to X$, show that the conjugate $\sigma \circ \tau \circ \sigma^{-1}$ of this cycle is again a cycle of length m, namely the cycle

$$\sigma \circ \tau \circ \sigma^{-1} = (\sigma(k_1), \sigma(k_2), \dots, \sigma(k_m))$$

(25 points)

Problem 2: We have seen in class that the centre $Z(G)$ of a group G is a subgroup of G. Show that it is normal. (25 points)

Problem 3: Suppose that G is a group with the property that every element $g \in G$ satisfies $g^2 = e$, where e is the neutral element of G. Show that G is abelian. (25 points)

Problem 4: Suppose that G is a group of finite even order. Show that there is a non-neutral element $g \in G$ such that $g^2 = e$, where e is the neutral element of G.

(Remark: This is a special case of Cauchy's theorem. The problem is not entirely easy.) (25 points)

Due date: Monday, September 25, 2023. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.