Memorial University of Newfoundland **Fall Semester 2023**
Yorck Sommerhäuser **MATH 3320: Sheet 1**

Abstract Algebra

Problem 1: Suppose that M and N are monoids. On the cartesian product $M \times N$, introduce the operation

$$(m, n)(m', n') := (mm', nn')$$

Show that $M \times N$ is again a monoid with this operation, called the direct product of M and N. Describe the identity element explicitly. (25 points)

Problem 2: Suppose that M is a monoid, and let

$$P := \{X \mid X \subset M\}$$

be its power set, i.e., the set of all subsets of M. For $X, Y \in P$, define

$$XY := \{xy \mid x \in X, y \in Y\}$$

(Note that XY is empty if X or Y are empty.) Show that P is again a monoid with this operation, and describe the identity element explicitly. (25 points)

Problem 3: Suppose that G is a group and that m and n are relatively prime integers. Suppose that g and h are two elements of G that satisfy

$$g^m = h^m \qquad \text{and} \qquad g^n = h^n$$

Show that $g = h$.

(Hint: You can use Bézout's lemma, which states that the greatest common divisor d of two integers m and n can be written in the form $d = mm' + nn'$ for two other integers m' and n'.) (25 points)

Problem 4: Suppose that $G = \{e, g, h\}$ is a group consisting of three different elements, where e is the identity element. Show that $g^2 \neq e$. (25 points)

Due date: Monday, September 18, 2023. Write your solution on letter-sized paper, and write your name on your solution. Write down all necessary computations in full detail, and explain your computations in English, using complete sentences. Prove every assertion that you make in full detail. It is not necessary to copy down the problems again or to submit this sheet with your solution.