

Tufts University
Department of Mathematics
Math 87 Midterm Project 1

Due: Thursday, October 17, at 10:30 a.m. (in class). Late projects will not be accepted!

In this problem, adapted from Vašek Chvátal's book *Linear Programming*, you are asked to maximize the net discounted revenue (NDR) over a decade for managing harvesting and planting in a forest. Currently, the forest is planted as follows:

Crop Type	Description	Acres	Volume if harvested
1	High-volume hardwoods	2754	2000
2	Medium-volume hardwoods	850	1200
3	Low-volume hardwoods	855	700
4	Conifer high forest	1598	4000
5	Mixed high forest	405	2500
6	Bare land	1761	

The last column tells us how many board-feet of lumber would be generated per acre by harvesting the trees. A secondary classification of areas with hardwood is based on how much undergrowth is present, as follows

	Complete Undergrowth	Partial Undergrowth	No Undergrowth
High-volume hardwoods	357	500	1897
Medium-volume hardwoods	197	130	523
Low-volume hardwoods	39	170	646

For any crop type, any number of acres can be treated in one of two ways: harvest and plant conifer (treatment 1A) or harvest and plant hardwood (treatment 1B). Bare land can also be treated similarly, planting conifer (treatment 1A) or hardwood (treatment 1B). Additionally, for hardwood areas with complete undergrowth, the trees may be harvested while retaining the undergrowth (treatment 2). For hardwood areas with partial undergrowth, the trees may be harvested while enriching the undergrowth (treatment 3). Finally, any number of acres of any type may be left alone (receive no treatment).

Net discounted revenue over the next ten years represents the present value of revenue for activities over the next decade. For this problem, the net discounted revenue (in dollars per acre) varies with crop type and treatment, and are given as follows:

Crop Type	1A	1B	2	3	None
1	287	215	228	292	204
2	207	135	148	212	148
3	157	85	98	162	112
4	487	415			371
5	337	265			264
6	87	15			61

Four additional requirements are imposed that:

- (i) The treated area must not exceed 5000 acres
- (ii) The resulting conifer area (equal to the untreated area of old conifer plus the area of newly planted conifer) must not exceed 3845 acres
- (iii) The volume of harvested hardwood must not exceed 2.44 million board feet
- (iv) The volume of harvested conifer and mixed high forest must not exceed 4.16 million board feet.

Your task is to formulate and solve (in matlab) a linear program to maximize the net discounted revenue of this problem, subject to the constraints explained above. You must

1. Draw a clearly labeled network-flow model for the linear program. You may not be able to include all constraints in this model, but explain which constraints you cannot include and why.
2. Use your network-flow model to write out the linear program. Be sure to use descriptive variable names.
3. Input your model into matlab and use the `linprog` command to solve it. Be sure to include a “legend” that explains the ordering of your descriptive variable names in the solution vector used by matlab. Include a printout of your setup and call to `linprog`, as well as the output.
4. Consider your model with the added constraint that at least 500 acres must be planted with hardwood - how does this change your answer?
5. Hand in a project report that is typed (aside from the network-flow model and resulting linear program, which may be hand-drawn and hand-written, respectively) and written in the style of a lab report and not that of a problem set. Fully explain how you formulated your linear programs (including all choices of nodes and edges). Write a summary of your solutions in economic terms, identifying the optimal treatment and harvesting plans and the net discounted revenue that will result from them.