Tufts University
Department of Mathematics
Math 50 Homework 3

Due: Thursday, October 6, at 1:30 p.m. (in class).

1. For the following linear programs, first sketch the feasible region and, then, find the values of x and y where the objective function obtains its maximum value. What is the maximum function value?

   (a) (20 points)

   $$\text{Maximize } 3x + y$$
   $$\text{subject to } 2x + y \leq 6$$
   $$x + 3y \leq 9$$
   $$x, y \geq 0$$

   (b) (20 points)

   $$\text{Maximize } 7x + 3y$$
   $$\text{subject to } 2x + y \leq 8$$
   $$x + y \geq 4$$
   $$x, y \geq 0$$

   (c) (20 points)

   $$\text{Maximize } 2x + 3y$$
   $$\text{subject to } x + y \geq 1$$
   $$y - x \leq 3$$
   $$2x + y \leq 9$$
   $$x - y \leq 3$$
   $$x, y \geq 0$$

2. (10 points)

   (a) Explain why a fair division between 2 people must also be envy-free.
   (b) Find a fair division amongst 3 people that is not envy-free. Note that this asks you to give both a set of tastes of 3 people and a division.

3. (10 points) Show that an equal division (where $d_{ij} = 1/M$ for all i, j) need not be Pareto-optimal. Note that this asks you to give a set of tastes and an alternate division that yields higher values.

4. (20 points) Download the files `cake_division.m` and `fair_division.m` from the course web-page (`http://neumann.math.tufts.edu/~scott/math87_F12`). Modify `fair_division.m` to generate a random 3×3 taste matrix and check if the division is fair, equitable, and envy-free. Run the code 1000 times, and report how many times the division found is fair and equitable, but not envy-free. Repeat this for a division between 4 people, using a random 4×3 taste matrix.

   Note that the following code generates a random 3×3 taste matrix.

```
T = rand(3,3);
for i=1:3,
    T(i,:) = T(i,:)/sum(T(i,:));
end;
```