**Tufts University**
**Department of Mathematics**
**Math 50 Midterm Project 1**

**Due: Tuesday, October 25, at 1:30 p.m. (in class).**

The Slick Petroleum Company owns two crude oil producing fields and operates two refineries. They sell a range of refined products that can be grouped into gasolines (of various octanes), light fuel oils (such as kerosene and diesel), heavy fuel oils (such as heating oil), and "salable residues" that are used in manufacture of asphalts, waxes, and other petrochemical products.

Their operations are as follows. Crude oil is transported via pipelines and tankers from the oilfields to the refineries, or it is sold on the open market to other companies (at the going market price). The refineries convert the crude into various "base products" that are blended into finished products and sold in six marketing areas. Refinery 1 (hereafter referred to as R1) supplies 5 nearby markets, numbered M1 through M5, while Refinery 2 supplies 3 nearby markets, M4 through M6 (so that M4 and M5 are supplied by both, but M1, M2, M3, and M6 have only 1 supplier). Market M1 is so far away from Refinery 1 (and even further from Refinery 2) that the company sometimes will buy products from the open market to supply it, rather than pay the high costs of shipping its own products.

Your task is to formulate and solve a linear program to maximize the profits of the Slick Petroleum Company, defined as the difference between the revenues and costs described below. Additionally, please make a recommendation to the company about how they can best increase the capacity at their refineries in order to make the largest gain in profit.

**Note:** Your project report must be typed and written in the style of a lab report and not that of a problem set. You must fully explain how you formulated your linear program (include figures if appropriate), present your linear program in the form of equations (be sure to pick identifying variable names), and discuss, in full sentences, the optimal solution of the linear program, as well as your advice on how to best increase the refinery capacity. This problem is a few years old, so the prices are out-of-line with modern costs. Feel free to speculate on how changes in the worldwide market for oil might update the solution to this problem.

## Supply and Transportation Costs

Crude oilfield C1 produces 30,000 bbl/day (barrels/day). This oil can be sold directly on the open market for \$3.50/bbl or be shipped, by tanker, to the refineries, costing \$0.30/bbl for shipment to R1 and \$0.60/bbl for shipment to R2.

Crude oilfield C2 produces 50,000 bbl/day. All the oil produced at C2 is transported by pipeline; up to 9,000 bbl/day to R1, at a cost of \$0.25/bbl, and up to 44,000 bbl/day to R2, at a cost of \$0.15/bbl. At the pipeline terminal at R2, oil can be used in R2, shipped by tanker to R1, at a cost of \$0.40/bbl, or sold to the open market, for \$2.60/bbl. Note that while C1 can sell directly to the open market from the oilfield, C2 must ship all of its oil via pipeline before selling any to the open market.

## Refining

Refinery R1 can process up to 40,000 bbl/day of crude oil, while R2 can process up to 20,000 bbl/day. The output of each refinery is a mixture of gasoline, light and heavy fuel oils, and salable residues that depends on which crude oilfield is used as supply and, in the case of R1, which process is used.

R1 is a modern refinery that offers two refining processes, a low gasoline yield process and a high gasoline yield process. These can be used in any proportion, meaning that any amounts of

crude oil from either field can be processed in either way, with the only limit being that not more than the equivalent of 25,000 bbl/day of crude oil from C1 can be processed in the high gasoline yield process. For the purposes of this limited capacity, a barrel of crude oil from C2 counts as 1.4 barrels of crude oil from C1. R2, in contrast, is older, and has fixed production processes for both C1 and C2 crude.

The percentage yields per barrel of oil processed in each possible way, as well as the processing costs are listed below.

| Percent Yields | Refinery R1 | | | | Refinery R2 | |
|---|---|---|---|---|---|---|
| | Low Gasoline Process | | High Gasoline Process | | | |
| | C1 | C2 | C1 | C2 | C1 | C2 |
| gasoline | 49 | 35 | 67 | 59 | 54 | 39 |
| light fuel | 15 | 14 | 19 | 18 | 14 | 16 |
| heavy fuel | 29 | 40 | 10 | 15 | 25 | 35 |
| salable residues | 7 | 10 | 3 | 6 | 6 | 8 |
| waste | 0 | 1 | 1 | 2 | 1 | 2 |
| Processing cost ($/bbl input) | 1.40 | 1.60 | 1.70 | 2.10 | 1.60 | 1.70 |

## Sales

All salable residues that are produced are immediately used again in other processes at the refineries. For these, net revenue at R1 is $1.80/bbl, while net revenue at R2 is $2.10/bbl.

Each marketing area has fixed demands for gasoline, light fuel oil, and heavy fuel oil, that must be met exactly. These are:

| Marketing Area | Gasoline | | Light fuel oil | | Heavy fuel oil | |
|---|---|---|---|---|---|---|
| | Demand bbl/day | Price $/bbl | Demand bbl/day | Price $/bbl | Demand bbl/day | Price $/bbl |
| M1 | 1800 | 9.50 | 400 | 5.60 | 0 | - |
| M2 | 9900 | 8.20 | 3300 | 4.80 | 4200 | 3.20 |
| M3 | 3200 | 9.10 | 600 | 5.40 | 0 | - |
| M4 | 6100 | 8.20 | 2200 | 5.00 | 3200 | 3.60 |
| M5 | 4200 | 8.60 | 900 | 5.20 | 800 | 3.80 |
| M6 | 6800 | 8.20 | 2600 | 4.80 | 3800 | 3.30 |

Additionally, heavy fuel oil can be sold to the open market directly from the refineries, with up to 6000 bbl/day sold at R1 at a price of $2.50/bbl and up to 2000 bbl/day sold at R2 at a price of $2.70/bbl.

The transportation costs to ship to each market area from each refinery are summarized below.

| Refinery | Marketing Area | | | | | |
|---|---|---|---|---|---|---|
| | M1 | M2 | M3 | M4 | M5 | M6 |
| R1 | 0.80 | 0.20 | 0.40 | 0.30 | 0.40 | - |
| R2 | - | - | - | 0.25 | 0.30 | 0.20 |

Because of the high costs of shipping to M1 from R1, gasoline and light fuel oil for this area may be supplied by buying directly from the open market, at a cost of $8.80/bbl for gasoline and $5.20/bbl for light fuel oil.