Tufts University
Department of Mathematics
Math 50 Homework 1

Due: Tuesday, September 20, at 1:30 p.m. (in class).

1. For a given population of whales, an estimate of the annual growth rate of the population without harvesting is given by $rx(1 - x/K)$, where $r = 0.08$ is the intrinsic growth rate, $K = 400,000$ is the maximum sustainable population, and x is the current population. (Notice that if $x = 0$ or $x = K$, the annual growth rate is zero, if $0 < x < K$, the annual growth rate is positive, while if $x > K$, the annual growth rate is negative.) If E boat-days of whaling are allowed per year, the annual growth rate is lowered by the amount $0.00001Ex$ (meaning that, for a population of x whales, each boat harvests 0.001% of the population for each day that it is whaling). For fixed E, the whale population reaches a steady state where the annual growth rate (including harvesting) is zero.

   (a) (15 points) How many boat-days of whaling per year should be allowed to maximize the steady-state harvest rate if the initial whale population is 70,000? What is the steady-state whale population? Plot the harvest rate and population size as functions of E.

   (b) (10 points) How sensitive is the optimal number of boat-days allowed to the value of r? How sensitive is the steady-state whale population to the value of r? Explain why your answers make sense.

   (c) (15 points) Suppose that instead of optimizing the steady-state harvest rate, we seek to maximize the steady-state annual profit from whaling, given that the cost of whaling is \$500 per boat-day and that each whale harvested can be sold for \$6000. How many boat-days of whaling per year should be allowed to maximize the total annual profit? What is the steady-state whale population? Plot the total profit as a function of E.

   (d) (10 points) How sensitive is the optimal number of boat-days of whaling to the cost per day? Would a large increase in the cost per day of whaling (arising from imposing a high fee for a whaling license) have a significant effect on the steady-state whale population? (Fully explain your answer.)

2. In this problem, you will develop the technique of *linear regression* to fit data. For parts (b)i, (b)ii, and (c), you should hand in some short written comments, as well as the plots produced by your code. For part (b), you should include a print-out of your function.

   (a) (15 points) Given n data points (x_i, y_i), find the slope, m, and y-intercept, b, of the line $mx + b$ that minimizes

   $$\sum_{i=1}^{n} (y_i - (mx_i + b))^2 .$$

   This line is said to be the "best fit" or "least-squares" fit line, since it minimizes the sum of the squares of the error between the data points and the data points.

   (b) (15 points) Write a program that, when given the data set $\{(x_i, y_i)\}_{i=1}^{n}$ as input, finds the least-squares fit line, and plots both the original data points and the line. Note that matlab already has such a function, `polyfit`, but you cannot use this (or similar functions that you did not write yourself) in your program.

i. Test your program with the data points $(1, 1)$, $(2, 4)$, and $(3, 9)$. What values of m and b do you compute? What is the least-squares error?

ii. Test your program with the data points $(1, 1)$, $(2, 2)$, and $(3, 3)$. What values of m and b do you compute? What is the least-squares error?

(c) (20 points) Download the data set from the course webpage (`http://neumann.math.tufts.edu/~scott/math50`) and find the least-squares fit of the data given there. This data gives the closing NASDAQ stock price of Yahoo! Inc. on each day that the market was open in 2010. The data is formatted into 3 columns: for each row, the first column gives the month for the data, the second column gives the day of the month for the data, and the third column gives the closing stock price on that day. You need to first convert the month-day data to a count of the days in the year; for this, you may want to explore the `datenum` function in matlab. What does your fit say about the general trend in the stock price over the year?