

Math 250-01

Numerical Methods for Partial Differential
Equations

Spring 2013

Course Information

BLOCK: G, MWF, 1:30-2:20 PM

INSTRUCTOR: Scott MacLachlan

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OFFICE HOURS: (Fall 2012) TF 11:00 AM - 12:30 PM

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PREREQUISITES: Math 135 and 151/251 or consent.

TEXT: No required text. Course notes will be given as handouts.

COURSE DESCRIPTION: While partial differential equations (PDEs) naturally arise in the mathematical models for many areas of science and engineering, analytical methods for the solution of these equations are successful only in certain special cases. For PDE models of heterogeneous media (for example, of flow through a porous aquifer, seismic imaging of the Earth's structure, or the electrical activation of the heart muscle), very little can be done with analytical techniques. Instead, we turn to computational simulation, which has become an increasingly important tool in many fields of science and engineering, replacing expensive or impractical experimentation in these areas. The focus of this course is on the key step of the simulation process, where approximations to the solutions of these PDEs are found using computers.

The course material breaks roughly into thirds. We will consider the numerical solution of hyperbolic equations using finite-difference discretizations. For these problems, we will look at the questions of accuracy, consistency, and stability, and how they are related through the Lax Equivalence Theorem. For elliptic equations, we will consider the finite-element method and its analysis using variational formulations and approximation theory in Sobolev spaces. Finally, we will see the connection between PDEs and the solution of large systems of linear equations, and the numerical techniques needed to do this efficiently. In particular, we will see how multigrid methods naturally arise in the numerical solution of elliptic equations and how they can be used to achieve optimally scaling simulation algorithms.

This course is intended to be approachable for students both from mathematics and from areas of science and engineering where computer simulation is commonplace. We will both prove mathematical theorems and write simulation codes.