Tufts University
Department of Mathematics
Math 250-03 Homework 1

Due: Thursday, September 20, at 3:00 p.m. (in class).

1. Solve the following ODEs. You may use any method you like, but must show your work; you may use any textbooks you like, but must do the solution "by hand".

   (a) (20 points)
   $$\begin{cases} \frac{dx}{dt} + 3x = t & t > 0 \\ x(0) = 1 \end{cases}$$

   (b) (20 points)
   $$\begin{cases} \frac{d^2x}{dt^2} - 4x = 0 & t > 0 \\ x(0) = 2 \\ x'(0) = 0 \end{cases}$$

   (c) (20 points)
   $$\begin{cases} \frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = 0 & t > 0 \\ x(0) = 2 \\ x'(0) = 3 \end{cases}$$

   (d) (20 points)
   $$\begin{cases} \frac{d^2x}{dt^2} + 4x = 0 & t > 0 \\ x(0) = 3 \\ x'(0) = 4 \end{cases}$$

2. (20 points) Starting from the (constant coefficient) wave equation in 3D,
   $$u_{tt} - c^2 \Delta u = q,$$
   assume that the solution, $u(\mathbf{x}, t)$, is periodic in time with frequency ω,
   $$q(\mathbf{x}, t) = e^{\imath\omega t} p(\mathbf{x}),$$
   $$u(\mathbf{x}, t) = e^{\imath\omega t} v(\mathbf{x}).$$
   Derive a PDE for $v(\mathbf{x})$. (This PDE is known as the Helmholtz equation.)

   Next, assume that $v(\mathbf{x}) = A(\mathbf{x}) e^{\imath T(\mathbf{x})}$ (the Rytov decomposition), and derive a PDE for $A(\mathbf{x})$ and $T(\mathbf{x})$. If $\omega \to \infty$, what is the dominant term in this equation?