MATH 2260 (Ordinary Differential Equations I) — Fall 2014
Midterm Exam #1

1. (2 points each, no partial credit) For each of the differential equations given, determine the order of the equation and if it is linear.

   (a) $x^4 \frac{\partial y}{\partial x} + y - x^6 = 0$

   (b) $\frac{\partial^6 y}{\partial x^6} + \left(\frac{\partial y}{\partial x}\right)^2 = x$

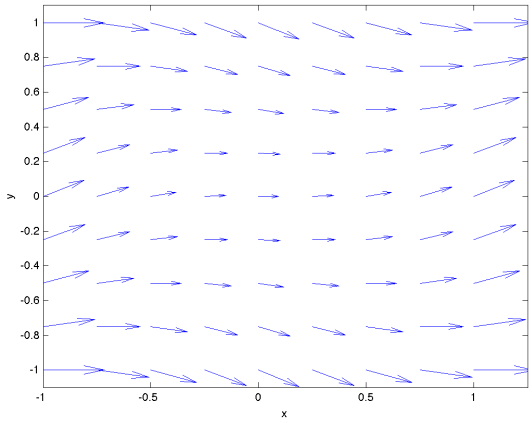
   (c) $x^6 \frac{\partial^3 y}{\partial x^3} + \sin(xy) = y^4$

   (d) $2x^2 y' + \ln(x) y = 0$

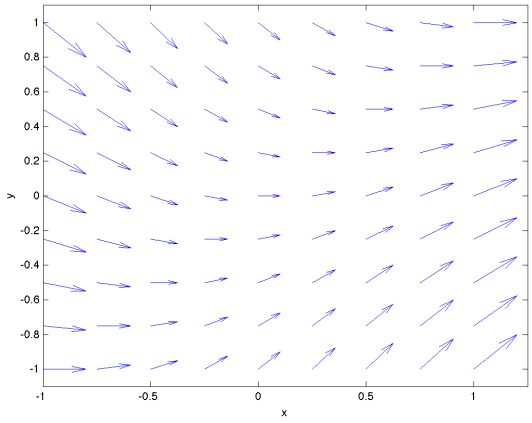
   (e) $y''' + xy'' + x^2 y' + xy^4 = 0$

2. (10 points) For each of the direction fields given below, sketch some integral curves of the equation.

   (a) $y' = \frac{x^2 - y^2}{1 + x^2 + y^2}$



   (b) $y' = \frac{x - y}{1 + x^2}$



3. (20 points) Solve the initial-value problem, $\frac{dy}{dx} + xy = x$, $y(0) = \frac{1}{2}$.

4. (20 points) Solve the initial-value problem, $y\frac{dy}{dx} = -x^2$, $y(0) = -1.5$. For what values of x is this solution well-defined?

5. (20 points) Solve $\frac{dy}{dx} - \frac{1+x}{3x}y = y^4$.

6. (20 points) Show that one of the following two equations is exact, and that the other is not. Find an implicit solution of the exact equation

   (a) $3x^2 \sin(x)y + (e^x - x^3 \cos(x))\frac{dy}{dx} = 0$.

   (b) $(ye^{xy}\tan(x) + e^{xy}\sec^2(x) + 2x)\,dx + (xe^{xy}\tan(x) + e^y)\,dy = 0$.

MATH 2260 (Ordinary Differential Equations I) — Winter 2015
Practice Midterm Exam #1

1. (20 points) Solve $2x\frac{dy}{dx} + \cos^2(y)\ln(x) = 0$.

2. (20 points) Solve $x^2\frac{dy}{dx} - 3x^2y = x^3$.

3. (10 points) Show that $(y - x)\frac{dy}{dx} - 7y = xe^{y/x}$ is a homogeneous nonlinear equation.

4. (20 points) Solve $\frac{dy}{dx} - \frac{1}{7}y = x/y^6$.

5. (10 points) Is $y(x) = xe^{2/x}$ a solution of $x^3y'' + 2xy' - 2y = 0$?

6. (20 points) Consider $(2x^2 + y)dx + (x^2y - x)dy = 0$

   (a) Show that the equation is not exact.

   (b) Find an integrating factor to make it exact.

   (c) Find an implicit definition of the solution $y(x)$.