MATH 2260 (Ordinary Differential Equations I) — Winter 2015
Homework #8

Due Date: Wednesday, April 1st, in class or in marking box #31 by 5:00 PM. You must show all work to receive credit.

1. (10 points) Solve the linear system using Cramer's rule

$$\begin{aligned} c_1 + c_3 &= 4 \\ c_1 + 2c_2 - c_3 &= -6 \\ c_1 + 4c_2 + c_3 &= -4 \end{aligned}$$

2. (10 points each) Find the particular solutions for the following ODEs

   (a) $L[y] = x^3y''' - 3x^2y'' + 6xy' - 6y = 2x$. For $x > 0$, the general solution of $L[y] = 0$ is $c_1x + c_2x^2 + c_3x^3$.

   (b) $L[y] = x^4y^{(4)} + 6x^3y''' + 2x^2y'' - 4xy' + 4y = 12x^2$. For $x > 0$, the general solution of $L[y] = 0$ is $c_1x + c_2x^2 + c_3/x + c_4/x^2$.

3. (5 points each) Using the definition, compute the Laplace transforms of the following functions.

   (a) $f(t) = te^{3t}$

   (b) $f(t) = e^t \sin(2t)$

4. (5 points each) Compute the Laplace transforms of the following functions. (You need not use the definition, but must show all work to receive credit.)

   (a) $f(t) = \cosh(\lambda t)$

   (b) $f(t) = t^2 - 7 + \cos 2t$

   (c) $f(t) = e^{2t+3}$

   (d) $f(t) = \sin\left(t + \frac{\pi}{6}\right)$

5. (10 points each) Solve the following equations using Laplace transforms. **NO credit** will be given for solutions that do not use Laplace transforms!

   (a) $D^2x - 2Dx = 4$, $x(0) = -1$, $x'(0) = 2$

   (b) $Dx - x = 2\sin(t)$, $x(0) = 0$

   (c) $D^2x + 2Dx + 2x = 25te^t$, $x(0) = x'(0) = 0$

   (d) $D^2x - x = \begin{cases} t & \text{if } t < 1 \\ 0 & \text{if } t \geq 1 \end{cases}$, $x(0) = x'(0) = 0$