MATH 2260 (Ordinary Differential Equations I) — Winter 2015
Homework #4

Due Date: Friday, February 20th, in class or in marking box #31 by 5:00 PM. You must show all work to receive credit.

1. (5 points each) Compute the Wronskian $W[y_1, y_2](x)$ for each pair of functions
   (a) $y_1(x) = e^{r_1 x}$, $y_2(x) = e^{r_2 x}$, $r_1 \neq r_2$
   (b) $y_1(x) = \cos(\omega x)$, $y_2(x) = \sin(\omega x)$, $\omega \neq 0$.
   (c) $y_1(x) = 1$, $y_2(x) = e^{rx}$, $r \neq 0$.
   (d) $y_1(x) = 1$, $y_2(x) = x$.
2. (5 points each) Find the general solution of the following ODEs
   (a) $y'' - 4y = 0$ (b) $y'' - 4y' + 3y = 0$ (c) $y'' - y' = 0$ (d) $y'' - y' - y = 0$
3. (10 points)
   (a) Verify that $y_1(x) = e^{2x}$, $y_2(x) = e^{5x}$ are solutions of $y'' - 7y' + 10y = 0$ for all x.
   (b) Verify that, for any c_1 and c_2, $y(x) = c_1 y_1(x) + c_2 y_2(x)$ is also a solution.
   (c) Solve the initial value problem with $y(0) = k_0$, $y'(0) = k_1$.
4. (10 points) Show that x^3 and $|x|^3$ are linearly independent for $-\infty < x < \infty$, but that they are not for $-\infty < x < 0$.
5. (10 points) Check that $y_1(x) = \sin(2x)$ and $y_2(x) = \sin(x)\cos(x)$ are solutions of $y'' + 4y = 0$ for $-\infty < x < \infty$. Is $y(x) = c_1 y_1(x) + c_2 y_2(x)$ the general solution of the equation?
6. (10 points) Check that $y_1(x) = x^{-1}$ and $y_2(x) = x^{-2}$ are solutions of $x^2 y'' + 4xy' + 2y = 0$ for $0 < x < \infty$. Is $y(x) = c_1 y_1(x) + c_2 y_2(x)$ the general solution of the equation?
7. (10 points) Let $y_1(x) = x^2$ and $y_2(x) = x^4$.
   (a) Compute $W[y_1, y_2](x)$.
   (b) Find an x_0 such that $W[y_1, y_2](x_0) = 0$. Find an x_1 such that $W[y_1, y_2](x_1) \neq 0$.
   (c) Is there any ODE $L[y] = y'' + p(x)y' + q(x)y = 0$ with continuous $p(x)$, $q(x)$ such that $L[y_1] = L[y_2] = 0$?
8. (10 points)
   (a) Show that $y_1(x) = 1$ and $y_2(x) = \sqrt{x}$ are solutions of $yy'' + (y')^2 = 0$ for $x > 0$.
   (b) Are $y_1(x)$ and $y_2(x)$ linearly independent for $x > 0$?
   (c) Show that $y(x) = c_1 + c_2\sqrt{x}$ is not a solution of the equation for all c_1 and c_2.
   (d) Why does this not contradict the theory discussed in class?