MATH 2260 (Ordinary Differential Equations I) — Winter 2015
Homework #3

Due Date: Wednesday, February 4th, in class or in marking box #31 by 5:00 PM. You must show all work to receive credit.

1. (5 points each) We say that an equilibrium solution, p_e, of $p'(t) = f(p)$ is "asymptotically stable" if there is an interval $a < p_e < b$ such that solutions for all initial values, $a < p_0 < b$ satisfy $\lim_{t\to\infty} p(t) = p_e$. Note that if $f(p) > 0$ for $a < p < p_e$ and $f(p) < 0$ for $p_e < p < b$, then p_e must be asymptotically stable. For each of the systems below, give all equilibrium solutions and state which are asymptotically stable.

   (a) $p' = r(1 - p/K)p$, $K > 0$

   (b) $p' = -r(1 - p/T)(1 - p/K)p$, $0 < T < K$

   (c) $p' = p(1 - p^2)$

   (d) $p' = ap - b\sqrt{p}$, $a, b > 0$

2. (10 points) The Schaefer model of a population subject to harvest says that, for a given rate of effort, E, to harvest a species, the population satisfies

$$\frac{dp}{dt} = r(1 - p/K)p - Ep.$$

   Assuming $E < r$, find the two equilibrium solutions of this equation. Which is asymptotically stable? The *sustainable yield* of the harvest is given by $Y = Ep_e$ for the asymptotically stable equilibrium, p_e. What choice of E maximizes Y?

3. (10 points each) For each of the following equations, determine if it is exact. If it is, solve the equation. If not, find an integrating factor and solve the resulting equation.

   (a) $(x + y)^2 dx + (x + y)^2 dy = 0$

   (b) $(3x^2 + 2xy + 4y^2)dx + (x^2 + 8xy + 18y)dy = 0$

   (c) $(\sin x - y \sin x - 2\cos x)dx + \cos x dy = 0$, $y(0) = 1$

   (d) $\left((e^x(x^2y^2 + 2xy^2) + 6x\right) dx + (2x^2ye^x + 2)dy = 0$

   (e) $2y^3 dx + 3y^2 dy = 0$

   (f) $-y dx + (x^4 - x)dy = 0$

   (g) $(a\cos(xy) - y\sin(xy))dx + (b\cos(xy) - x\sin(xy))dy = 0$