MATH 2260 (Ordinary Differential Equations I) — Fall 2014
Midterm Exam #2 Review problems

Section 5.1, #2,5
Section 5.2, #2,4,6,8,10,12,14,16
Section 5.4, #2,4,6,8,10,12,14,16,18,24,26,28
Section 5.5, #2,4,6,8,10,22,24,26,28
Section 5.6, #18,22,24,26,28,30
Section 9.1, #4,6,10
Section 9.2, #2,4,6,8,10,12,16,20,28,30,32,34,36,38
Section 9.3, #2,4,6,8,...,60,62,64,66

Other problems:

1. Check that $y_1(x) = \cos(2x)$ and $y_2(x) = 4\sin^2 x - 2$ are solutions of $y'' + 4y = 0$. Do they form a fundamental set of solutions?

2. Check that $y_1(x) = 2$, $y_2(x) = x^2$ are solutions of $xy'' - y' = 0$ for $x > 0$. Do they form a fundamental set of solutions?

3. Check that $y_1(x) = e^{-x}$, $y_2(x) = x+3e^{-x}$ and $y_3(x) = x$ are solutions of $(D^3+D^2)y = 0$. Do they form a fundamental set of solutions?

4. Find all solutions of the form x^α to $(x^2D^3 + 2xD^2 - 2D)y = 0$ for $x > 0$. Do they form a fundamental set of solutions?

5. (a) Show that $y(x) = c_1\cos(x) + c_2\sin(x) + c_3e^x$ is not the general solution of $(D^4 - 1)y = 0$ by finding a set of initial conditions, $y(0) = k_0$, $y'(0) = k_1$, $y''(0) = k_2$, $y'''(0) = k_3$, that cannot be matched by a function of this form.

   (b) Find the true general solution, and the solution which matches your initial conditions in part (a).

MATH 2260 (Ordinary Differential Equations I) — Fall 2014
Practice Midterm Exam #2

1. (20 points) Find the general solutions of the equations below

   (a) $(D-1)^2(D+2)y = 0$

   (b) $(D-2)(D^2+2)y = 0$

   (c) $3(D^2+D+2)^2y = 0$

   (d) $(D-1)(D-2)^2(D+3)^3y = 0$

2. (20 points) Solve the initial value problem $4y'' - 4y' + 101y = 0$, $y(0) = -4$, $y'(0) = 13$.

3. (5 points) Determine a second-order linear homogeneous equation with constant coefficients for which $y(x) = 7xe^{-4x}$ is a solution.

4. (30 points) One solution of the equation $4x^2y'' + 8xy' + y = 0$ for $x > 0$ is $y_1(x) = \frac{1}{\sqrt{x}}$. Use the method of reduction of order to find a distinct second solution, $y_2(x)$, to the equation. Use the Wronskian to prove that $\{y_1, y_2\}$ is a fundamental set of solutions to the equation.

5. (15 points) Find the general solution of $y'' + 4y = e^x + 1$.

6. (10 points) Make a guess for the form of a particular solution to

$$(D+2)^7(D^2+1)^6y = xe^{-2x} + \sin(x).$$

   Do not evaluate the coefficients in the guess. No credit will be given for guesses that include solutions of the related homogeneous equation.